SINGLE-PHASE COMMUTATOR MOTORS

ELECTRIC MECHANISM
PART I.

SINGLE-PHASE COMMUTATOR MOTORS

BY

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To

MY FATHER,

WITHOUT WHOSE ENCOURAGEMENT THIS
RESEARCH WOULD NEVER HAVE
BEEN COMPLETED.

PREFACE

Every reader of the current literature on single-phase motors must feel an impression of the artificiality of the present methods of study, and their remoteness from all physical conceptions of what goes on in these machines. When reading these articles one feels oneself in a mathematical world, quite apart from reality, and cannot help entertaining a conviction that there must be *some* simpler way of putting the matter. The writer has himself published articles of this artificial character, based on the old conceptions, but is obliged to admit that they are of very little practical help to the engineer, and give very little insight into the phenomena really taking place. It is needless to say that a clear understanding of the phenomena is a necessary preliminary to improvements in practical application.

It gradually became clear to the writer that a radically new departure was necessary, to eliminate, on the one hand, the complex analysis hitherto necessary, which quite prevents a clear grasp of the physical facts, and, on the other, to lead direct to the results the designer requires, without artificiality and circumlocution. We require a theory which can readily be reduced to a numerical form, not one that merely gives us vague general ideas and complicated formulas.

The present treatise is an attempt to supply this want. Whether successful or not, must be left to the reader.

One may compare the state of affairs in single-phase motor work at the present day to the state of affairs in poly-phase motor work before the discovery of the circle diagram about 1895. Reading old articles on the subject, published before that date, one is surprised that a problem which we know to be fairly simple could be treated in so complicated and unnatural a way. All these old methods have now been swept away, and the action

of the induction motor can now be explained in a clear and simple manner.

It is the same with the single-phase motor, and it is believed that the methods of the present volume furnish a ground-work on which a clear and simple general theory may be built up so as to be intelligible to any earnest reader.

It has been found necessary, as will be seen in Chapter III., which contains the fundamental principles of the methods employed, to abandon the phase diagram and build up a new vector diagram, in which the directions of vectors represent directions in space and not in phase. The volume may, therefore, present difficulties, owing not to the intrinsic complication of the subject, but to the novelty of the fundamental ideas. If, in writing on the subject of ordinary alternating current phenomena, it were necessary to assume the reader quite unacquainted with the ordinary phase diagram, and to develop it with all accompanying ideas and conventions from first principles, the simplest investigations would take on an appearance of complexity which is really quite foreign to them.

Yet this is what we have been obliged to do in the present volume, and any difficulties which the reader may find will probably be due, at least as much to unfamiliarity with the novel ideas involved, as to the real difficulties of the subject, which will be found sufficiently simple as soon as the reader has thoroughly grasped the principles of the methods which are used.

The development of a new method of alternating current analysis, independent of all others, with a suitable notation and conventions to accompany it, is no easy task, and has involved an amount of labour quite out of all proportion to the size of the volume. Yet, nevertheless, it may well be that points of difficulty still remain in the exposition, as well as unsolved problems. This is unavoidable in a new subject, and it is the author's hope that other writers may take the matter up and remove some of these difficulties.

The present volume may be expected to be useful to two classes of reader:—

(1) To those professionally interested in the type of machine with which it deals.

(2) To the advanced technical student and teacher.

Many technical colleges are equipped with a good supply of single-phase motors, but these are almost useless, because a comprehension of their action is quite outside the limits of an ordinary college course. By the methods given in the last chapter of the present volume, however, one may very readily plot curves of field distribution by direct experiment, without any knowledge of the theory of the subject, and, in this way, a very clear idea of their operation may be simply obtained. I have received material assistance in revising the proofs from my friends Messrs. F. M. Denton, R. T. Looser, and J. T. Irwin. I have also to thank the American Institute of Electrical Engineers for permission to reproduce a portion of my paper on the "Shunt Induction Motor" appearing in their Transactions for 1909.

F. Creedy.

CONTENTS

SINGLE-PHASE COMMUTATOR MOTORS

CHAPTER I

THE CHIEF TYPES OF SINGLE-PHASE MOTOR

In the present treatise an attempt will be made to present the theory of the single-phase, or what we shall learn, more generally, to term the elliptic-field motor in a manner sufficiently general and complete to enable the careful reader to calculate, with some approach to accuracy, both the characteristics of any such machine with varying load, and the distribution of current, flux, and E.M.F. within it. The number of possible variations of connection is unlimited, and, of course, only a few leading types can be treated of, but it is hoped that this may be done in such a way as to expose the principles, and place the reader in a position to calculate the characteristics of any connection which is proposed to him. The subject is relatively a difficult one and will always remain so. It is not, however, nearly so much so as one might suppose from certain articles appearing from time to time in the technical journals.

Two types of single-phase commutator motor exist—the series and the shunt type—having characteristics somewhat analogous to series and shunt continuous-current motors.

We shall first discuss the series type. In all single-phase motors two distinct problems arise:

(1) What are the characteristics of the machine with varying load?

(2) What distribution of current, flux, and E.M.F. exists in it at each speed?

The user requires an answer to the first, and the designer to the second question.

The first question has been very exhaustively discussed by a multitude of writers and accordingly we shall treat it rather briefly, giving references to other writers for fuller treatment, and devoting ourselves chiefly to the second question, which has been much less discussed.

Diagrammatic Notation for the Circuits and Connections of a Dynamo-Electric Machine.

During the last ten years or so, dating, I believe, from Atkinson's famous paper, a diagrammatic method which we shall frequently have occasion to employ, has grown up for indicating the connections of dynamo-electric machines. In this notation a zig-zag line is used to represent a stator winding having the axis shown dotted. We shall assume this winding harmonically distributed. A circle is used to represent a rotor and its winding

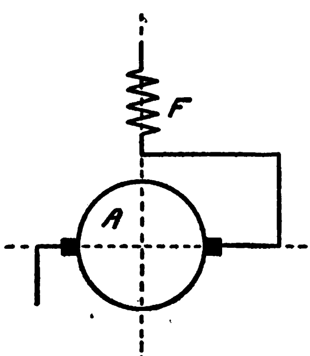


FIG. 1.

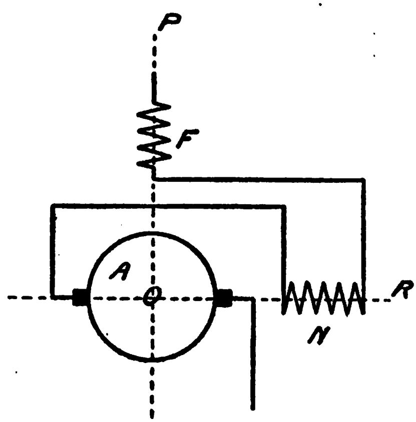


FIG. 2.

connected to a commutator, while small rectangles on the circumference of the circle represent the brushes. Any current flowing through the brushes is assumed to produce a magnetomotive force along the line joining them, shown dotted. Thus the diagram (Fig. 1) is that of a simple series motor, without neutralizing coil.

The neutralized series motor will be as shown in the next figure (Fig. 2).

The series-type motor takes five principal forms:

(1) The Neutralized Series Motor.

The neutralized series motor (Fig. 2) consists of a field winding F placed upon the stator and connected in series with a commutator armature A, and a neutralizing coil N also wound on the stator, and so connected that its M.M.F. is as nearly as possible equal and opposite to that of the armature, and hence the two together are very nearly non-inductive. The neutralizing coil, if short-circuited upon itself will be almost as effective.

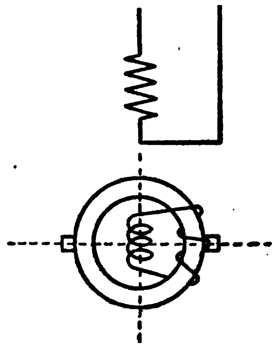


Fig. 3.

In connection with the series type motor, it will be desirable to discuss the effects of the commutating coil at starting on the characteristics of the machine, since they are more important in this type, perhaps, than in any other. We may regard the commutating coil as a short-circuited secondary to the coil F, of high resistance and negligible local self-induction (see Fig. 4).

Reducing this by the transformer diagram (Fig. 6) given on p. 4 we get Fig. 5, which is a repulsion motor, in which the

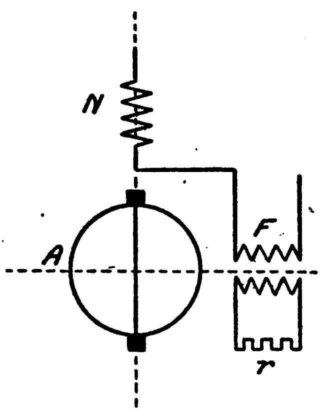


Fig. 4.

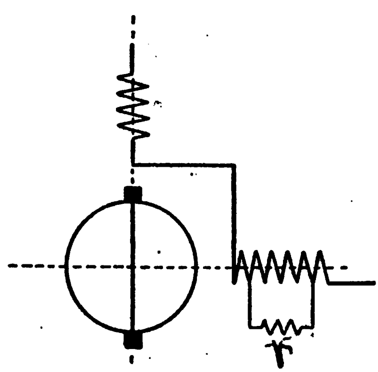


Fig. 5.

high resistance r merely shunts a portion of the winding F, which corresponds to the exciting impedance Z_0 in the transformer diagram, while the unshunted portion corresponds to the primary

leakage impedance Z_1. For practical purposes, however, we may assume that the resistance r shunts the whole of the winding F, whence we derive the following important rule:

The effect of the commutating coil at starting on the characteristics of the single-phase series type motor may be represented by that of a nearly non-inductive resistance shunting the winding which produces the flux in which the commutating coils lie.

A more elaborate treatment (Chap. IX.) is required for running conditions.

(2) The Induced Series Motor.

This machine (Fig. 7) consists of the same three elements A, F and N, as the neutralized series motor, but here N is joined across the line, while F is connected directly in series with A, the two being closed on themselves. By this means we make the rotor voltage independent of the line voltage.

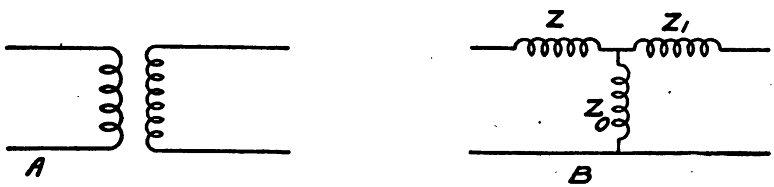


FIG. 6.

The machine has practically the same load characteristics as the neutralized series motor, although the internal flux distribution, etc., is entirely different. For the purpose of studying the load characteristics we shall make use of the well-known reduction of the transformer to a divided circuit, due, I believe, to Steinmetz (see Fig. 6).

The object of this reduction is to show how the effects of resistance and leakage in the transformer on the primary and secondary currents and on the secondary E.M.F. may be represented by a number of impedances suitably connected as shown in Fig. 6.

We may define Z, the impedance of a circuit, as the voltage across it, due to unit current in it. Defining Z in this way, in Fig. 6 Z is the voltage in the primary due to the resistance and the leakage flux combined, when unit current flows through it.

Z_1 is the same voltage when unit current flows through the secondary, and Z_0 is the voltage due to the main flux.

With these definitions, we may state the following theorem.

The current flowing into the primary of the one-one ratio transformer A (Fig. 6), the current flowing out of the secondary and the secondary E.M.F. will be identical with the current flowing into the divided circuit B (on the left), the current flowing out (on the right) and the voltage between lines (on the right).

This is true under any condition of load. The divided circuit B is made up of three impedance coils giving voltages Z, Z_1 and Z_0 respectively when traversed by unit current.

Applying this to the induced series motor, we get Fig. 8, which is identical with the neutralized series motor save that the impe-

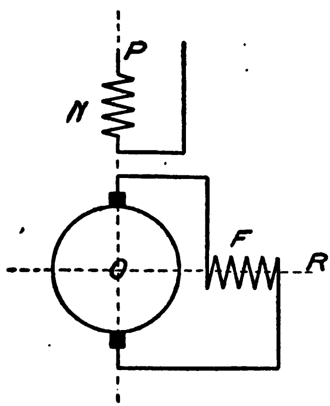


Fig. 7.

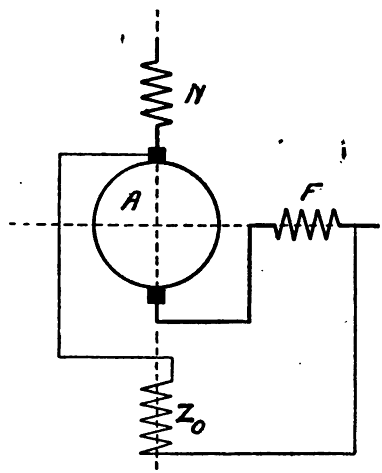


Fig. 8.

dance Z_0 is connected in series with the neutralizing coil across the line. It is convenient to represent this impedance Z_0 as a shunt coil wound upon the stator and producing identically the same flux as that due to the transformer or primary coil of the original motor. Thus by making use of this reduction of the transformer we show that to every inductive type of motor there corresponds a conductive type having, not approximately but exactly, the same characteristics, and the same flux distribution. Hence, in addition to the main current, the neutralizing coil

carries an extra magnetizing current $\frac{E\sigma}{Z_0 + Z}$ and the E.M.F. across the armature is accordingly somewhat less.

In addition to the main current corresponding to any load, which will be substantially the same as in the neutralized series motor, this machine takes a wattless magnetizing current, to excite the flux parallel to the brush line, which is required since the armature E.M.F. is induced rather than conducted in.

The power factor is hence somewhat less than that of the neutralized series motor.

(3) The Repulsion Motor.

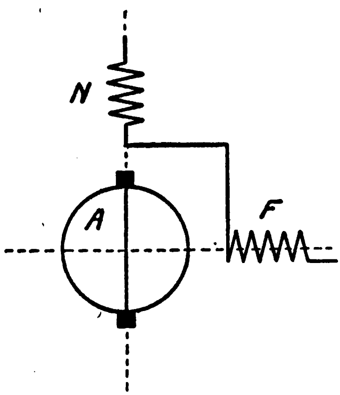


FIG. 9.

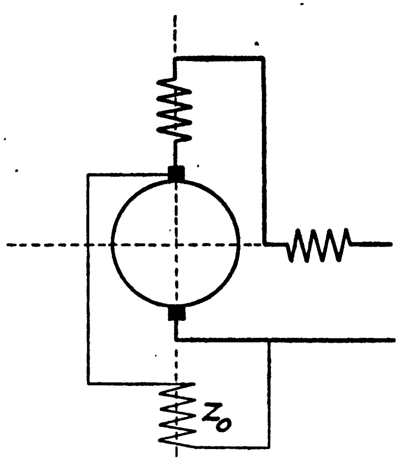


FIG. 10.

The repulsion motor may be regarded as another modification of the neutralized series motor formed by connecting the three elements A, F, and N in a third way. In this machine (Fig. 9) N is connected in series with F, and A is short-circuited.

Applying our transformer diagram as before, we get the equivalent conductive diagram of Fig. 10, in which the impedance Z_0 appears shunting the armature.

The field current is now no longer the same as the armature current, and will not be in phase with it.

(4) The Inverted Repulsion Motor.

The inverted repulsion motor is constructed in exactly the same manner as the ordinary repulsion motor, save that the line terminals are connected across the rotor while the stator is short-circuited as in Fig. 11.

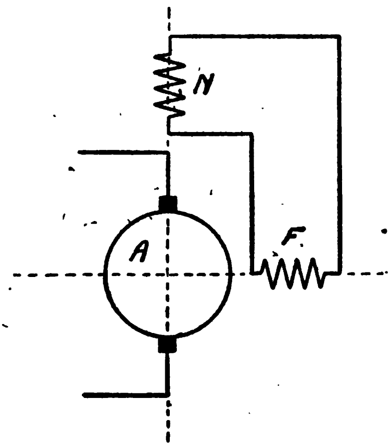


FIG. 11.

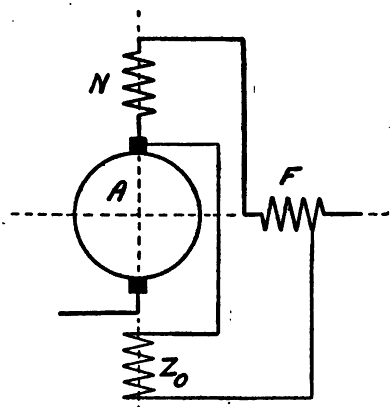


FIG. 12.

Reducing this by the transformer diagram (Fig. 6) as before, we see that the auxiliary impedance coil Z_0 is now placed in shunt to F and N; the machine, of course, being otherwise the same as the neutralized series motor.

(5) The Compensated Repulsion Motor.

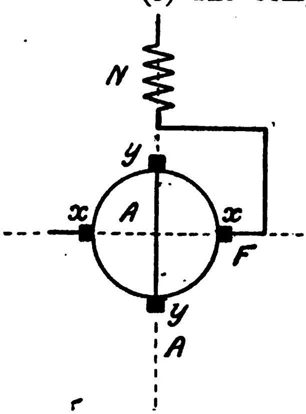


FIG. 13.

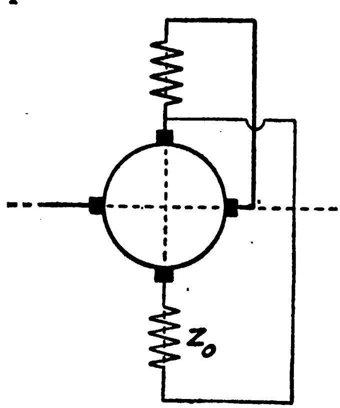


FIG. 14.

This type of machine is constructed as shown in Fig. 13. It bears four sets of brushes on the commutator per pair of poles, of which one pair is short-circuited and the other pair connected in series with the stator winding which is perpendicular to their axis, that is, parallel to the axis of the short-circuited brushes. If we compare this with the ordinary repulsion motor (Fig. 9), we see that it differs therefrom only in that the field winding F, instead of being on the stator, is on the rotor.

We may reduce this by the transformer diagram exactly as before, obtaining Fig. 14.

Shunt-Type Machines.

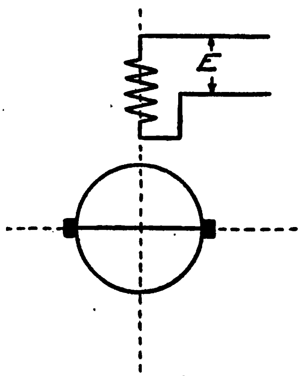


FIG. 15.

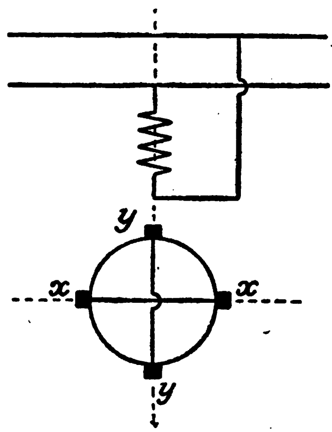


FIG. 16.

The shunt-type machines introduce a new element—the method of exciting the shunt field.

The ordinary shunt motor built with the armature and field in parallel is useless on alternating current, because the field flux lags 90° behind the E.M.F., while the armature current tends to come into phase with the E.M.F. and hence go out of phase with the flux as the speed rises. To prevent this phase displacement, which would destroy the torque, it is necessary to excite the field by means of an E.M.F. leading 90° on that applied to the armature. The method by which this is done without introducing another phase, invented by L. B. Atkinson, constitutes the principal peculiarity of the shunt-type motor.

Consider a commutator type armature (Fig. 15), fitted with a

pair of short-circuited brushes and revolving in a field excited by a coil whose axis is perpendicular to that of the rotor circuit. If the E.M.F. applied to this coil be E_0, the flux due to it will, of course, lag 90° behind the E.M.F. producing it. Owing to the rotation of the rotor in this flux, an E.M.F. will be induced in the rotor circuit, exactly as in a continuous-current machine, in phase with this flux and consequently in quadrature with the E.M.F. applied to the stator circuit. The rotor E.M.F. in its turn, excites a flux at right angles in space to that due to the stator circuit and lagging 90° behind the E.M.F. to which it is due. This rotor flux, therefore, is in phase with the terminal E.M.F.

It is by this device that the difficulty of obtaining a field flux in phase with the terminal E.M.F. is met, and it will be found that all single-phase shunt motors embody it.

The field flux, of course, is not constant as in the continuous-current shunt motor, but directly proportional to the speed, and is zero at standstill. Thus, single-phase shunt-type motors have little or no starting torque and have to be started as series-type motors. They are differentiated from the series-type motor by the fact that the flux is independent of the load, and hence they have constant speed characteristics, like the continuous-current shunt motor, though they differ from it so considerably in construction.

Before discussing the relations between the inductive and conductive types of shunt motor it will be desirable to give a brief description of some of the principal types.

All practical machines of this class are developments of the machine shown in Fig. 16, usually known as the Atkinson commutator induction motor. This consists of a slotted stator, or primary, with a uniform air-gap all round, equipped with a single distributed single-phase winding, and a drum-wound rotor fitted with a commutator, as in a direct-current machine. On the commutator rest four brushes, or groups of brushes (in a two-pole machine); the axis of one pair of brushes usually lies parallel to that of the primary winding and that of the other perpendicular thereto, but this position is not essential. Let us repeat the explanation given above of the action of this motor in slightly different words. At standstill the primary coil merely induces a current on the YY axis (Fig. 16), and hence there is no starting torque. But as the motor speeds up, a current is induced along

the XX axis which produces a flux along that axis. This current and the flux it produces are nearly in phase with the impressed E.M.F. and hence in quadrature with the flux along YY. Therefore, the flux along YY (the primary flux) can produce no torque in combination with the current along XX. In the neighbourhood of synchronism, however, the current along YY, or the load current, is nearly in phase with the flux along XX; hence there will be a torque due to these two in combination.

The flux along XX may properly be regarded as the "field" flux of the motor, since it is the flux to which the torque is due. Similarly the current along YY may be regarded as the armature current, since it has the same functions as this current in a direct-current machine, although, of course, it is induced from the stator, instead of being led directly in as in the continuous-current motor.

The E.M.F. on the XX axis is due to the movement of the rotor conductors through the primary flux, or flux interlinking the primary circuit. It is, therefore, directly proportional to the speed. The flux along the XX axis, which is determined by this E.M.F. in the same manner that the primary flux is determined by the primary E.M.F., is therefore also directly proportional to the speed.

Such a motor reaches its limiting speed when the counter E.M.F., which may be represented by E_2, induced by the field flux (the flux along the axis XX) in the armature circuit along the axis YY, balances the E.M.F. E induced therein by the primary flux; or, more exactly, when the vector difference, $E—E_2$, between these two is in quadrature with E_2, the counter E.M.F. This condition is the same as that which limits the speed of the direct-current shunt-wound motor. Hence it would seem that the same methods which are available for varying the speed of the latter should be available here. There are two of these methods: (1) variable voltage control and (2) adjustable field excitation. To apply the first method to the single-phase shunt induction motor we have merely to feed an extra voltage from the line into the YY axis, say, by means of a transformer, as shown in Fig. 17, thereby increasing E. In order that the increased E.M.F. E may be balanced by E_2, the motor must clearly run at a higher speed.

It might be thought at first sight that in order to increase E,

it is merely necessary to increase the primary flux, say, by increasing the primary impressed E.M.F. This, however, is a fallacy, because the counter E.M.F. E_2 is due to the rotation of the armature in the primary field and is therefore proportional to that field. Hence any variation of the primary field simply varies E and E_2 in the same proportion, and the speed consequently remains the same. One may also remark that the secondary field flux becomes effectively equal to the primary flux at synchronous speed, and hence E_2 is effectively equal to E at that speed, which is practically the free running speed of the normal motor.

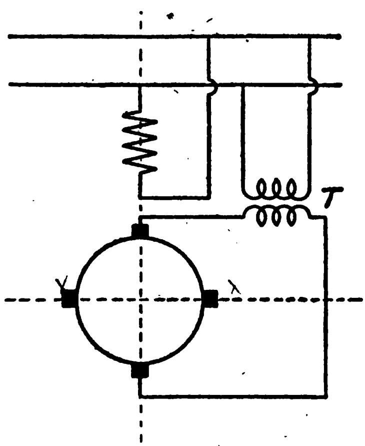


FIG. 17.

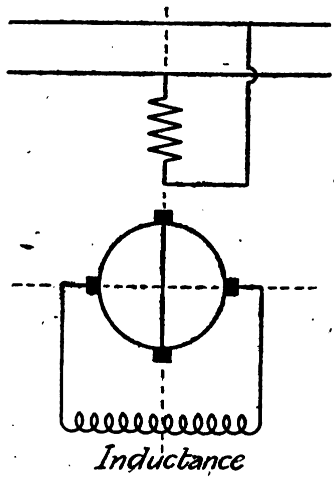


FIG. 18.

A second method is to weaken or strengthen the field along the XX axis by inserting an inductance or a capacity in the XX circuit, as shown in Fig. 18. An inductance tends to raise the speed and a capacity to lower it. This method is precisely analogous to the speed variation of shunt-wound direct-current motors by field control, except that inductance or capacity must be used instead of resistance. The cause of this is that the circuit XX is an almost purely inductive one, and consequently the magnetizing current which flows in it lags practically 90° behind the E.M.F. which produces it. It is not desired to alter the phase of this current, or of the flux it produces; hence in order to weaken it an inductance is inserted in the XX circuit,

and to strengthen it, a capacity. The use of resistance would tend to change the phase of the flux and impair the torque.

There is a third method which has no precise analogue in direct-current work. Clearly the flux necessary to balance the E.M.F. E_1 induced in the XX axis depends on the number of turns in the XX circuit. Hence if a coil be put on the stator with its axis parallel to the XX axis, and connected in series with the XX brushes (see Fig. 19), it will either strengthen or weaken the flux along the XX axis, according as the M.M.F. of this coil opposes or assists the rotor ampere-turns.

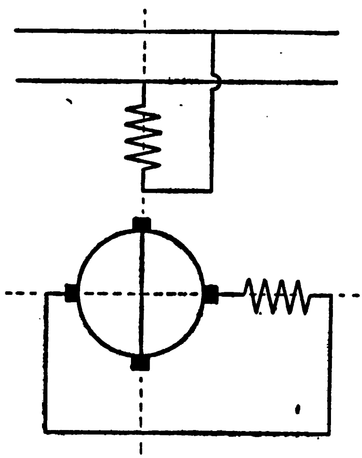


FIG. 19.

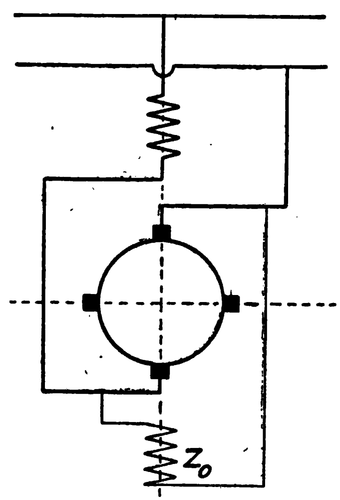


FIG. 20.

These three methods of speed variation are the only radically distinct methods which may be applied to the single-phase induction motor pure and simple.

By applying the transformer diagram in the same way as we did to the series motor, we can derive a set of shunt-conduction motors akin to the series-conduction motors we discussed above. For instance, applying our transformer diagram to the simple Atkinson commutator-induction motor of Fig. 16, we get Fig. 20, the shunt-commutator conduction motor.

These two machines are, from a theoretical standpoint, identical. They differ only constructionally. By feeding an auxiliary voltage into the armature, as in Fig. 21, without varying that

across the shunt coil, we can clearly vary the speed of the motor on just the same principle as that applied to the shunt-induction motor of Fig. 17. The other two methods of speed variation can clearly be applied to the conduction motor in their original forms.

Most of the practical forms of shunt-commutator motor make use of phase compensation. This will be discussed more fully

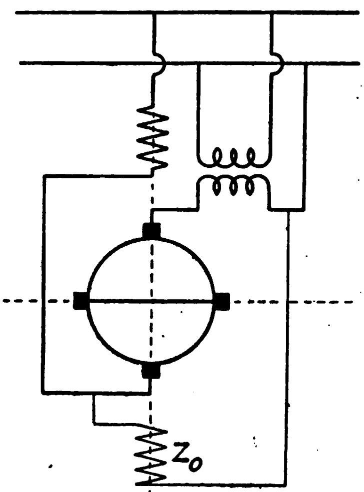


Fig. 21.

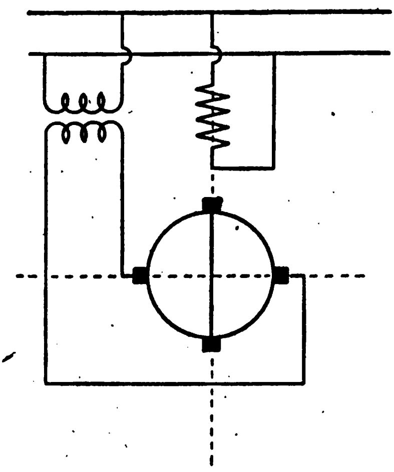


Fig. 22.

later on, but in the meantime it will be as well to give a diagram of the compensated shunt-induction motor. This is given in Fig. 22. It will be seen that this motor is the same as the Atkinson commutator-induction motor save that a voltage in phase with the terminal voltage is introduced into the pair of brushes at right angles to the primary axis.

CHAPTER II

CHARACTERISTICS OF SERIES TYPE MOTORS

We shall now attempt to answer the question, What are the characteristics of the different machines with varying load?

This question has been studied by a great number of writers, and has been fully worked out for most of the types of machine mentioned above. We shall not, therefore, devote a great deal of space to it, but merely give such a general discussion as may serve for all types. The method usually adopted in discussing this question has been the following:

We formulate a set of equations involving k, the speed, as independent variable, expressing the action of the motor. These equations are derived from the fundamental laws of the electric circuit. From them we derive an equation giving us the locus of the vector representing the current supplied to the motor, as the speed varies, the E.M.F. being assumed constant in magnitude and phase.

It is often possible, after this curve has been obtained, to derive from it constructions whereby the speed, torque, output, etc., of the machine under consideration may be obtained by a linear geometrical construction.

The locus of the current-vector is usually, though not always, a circle. Hence this method of study is often called the method of the circle diagram. While, of course, the final diagram is the same, or similar, in all cases, the method of deduction varies very widely among different authors. The present writer has usually found the "symbolic" methods of Steinmetz by far the most convenient, and they will accordingly be used in the present chapter where necessary. However, a great deal can be done without them.

The first and most important thing which we have to notice is that the relation between current and E.M.F. in the electric circuit is *linear*, that is, if the circuit constants or conditions are not changing, every current consumes an E.M.F. proportional to

itself. This is expressed algebraically by the *impedance equation* or generalized Ohm's law

$$e = iz \qquad (1)$$

z being called the impedance, a function of the circuit conditions alone.

It has been shown by Steinmetz that where e and i are simple harmonic functions of the time they may be represented, together with z, by ordinary complex quantities in the above equation, when it will express not only the magnitude but the phase relations also.

It follows from this equation that if i is constant and z varies, e will be directly proportional to z. Conversely, if z varies and e is constant, $i = \frac{e}{z}$ will be inversely proportional to z. Hence, in the case of any electric circuit in which the circuit conditions are changing and z, therefore, varying, we may draw two curves, one representing the variation of e on the hypothesis that i is constant, and the other that of i, on the hypothesis that e is constant. These curves will be reciprocal to one another. We shall make use of these facts in deducing our locus diagrams.

The Single-Phase Series Motor.

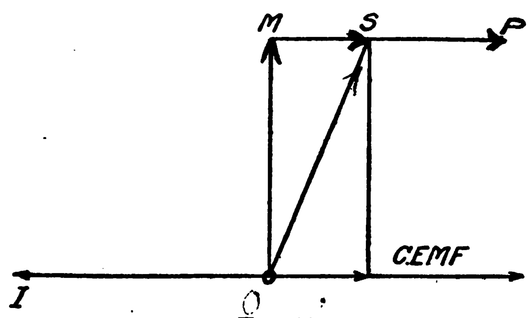


FIG. 23.

Let us consider the E.M.F. diagram in this machine when we keep the current constant and vary the speed, the machine running as a motor. At starting, the E.M.F.'s are as follows:

(1) Opposite in phase with the current, *ir* the resistance E.M.F. or MS in Fig. 23.

(2) in quadrature therewith, *iLp* the E.M.F. of self-induction

(where L is the co-efficient of self-induction of the whole machine and $p = 2\pi \times$ frequency) or OM in the diagram. OS is therefore the resultant back EMF produced by the current equal and opposite to the terminal E.M.F.

When the machine begins to run we get an additional counter E.M.F. SP, opposite in phase with the current and proportional to the speed, acting like an additional *ir* drop. The E.M.F. of self-induction remains invariable, of course. The locus of the E.M.F. vector is therefore a straight line parallel to that chosen for the current. If we reverse the motor, of course making it a generator, the counter E.M.F. becomes opposite in sign to the resistance drop, and at a certain speed cancels it, leaving nothing but the E.M.F. of self-induction, OM which is perpendicular to the locus of the E.M.F. vector. At this speed, current and E.M.F. are exactly in quadrature. Now, as we noted above, in order to find the locus of the current vector for a constant E.M.F. all that is necessary is to take the reciprocal of the vectors to the E.M.F. locus. We may note in passing that if OS be the E.M.F. vector at zero speed, and OP that at any other speed, then SP is directly proportional to the speed and the line SP can accordingly be calibrated in terms of the speed.

If we take the reciprocals of all such vectors as OP they will lie in a circle, as is well known geometrically. Hence—

In the single-phase series motor the locus of the current vector is a circle.

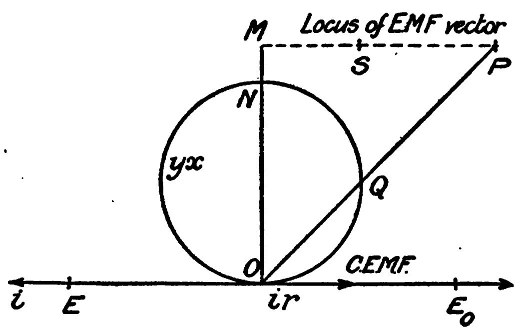


FIG. 24.

We must now determine what is the phase of our current vector relative to the (constant) E.M.F. as it moves on its circle. To ascertain this, consider the point M on the E.M.F. locus (Fig. 24)

where the E.M.F. of self-induction is the only one left in the circuit. Corresponding to this we now have $ON = \frac{1}{OM}$. Since at this speed current and E.M.F. are exactly in quadrature, and ON now represents the current at the speed in question, it follows that the E.M.F. of self-induction, which is now equal and opposite to the terminal E.M.F., must be represented by OE, lagging 90° on ON. The terminal E.M.F., of course, is OE_0 opposite to OE. In order to ascertain what speed corresponds to a given current OQ all that is necessary is to extend the current vector OQ till it cuts the calibrated E.M.F. locus in P. Then SP, as we saw above, is a measure of the speed.

The torque, of course, in the ideal motor which we are considering, in which saturation is avoided, is proportional to the square of the current, or OQ^2. Hence all the essential quantities which we desire to study are portrayed in the diagram.

By the use of various geometrical devices the representation of such quantities as the efficiency, output, etc., may be effected in a simple manner. We shall not stop to discuss these, but merely give references for their deduction.*

The Repulsion Motor.

We have seen that the repulsion motor is equivalent to a neutralized series motor with a shunt coil connected across the armature. Its diagram, therefore, will be similar to that of the series motor. First, let us assume the line or field-current constant. At starting, OS will be the E.M.F. corresponding to the constant current i. This may be calculated in the ordinary way by the principles of the divided circuit. As the machine speeds up a counter E.M.F. SP (Fig. 23), opposite in phase to the current and proportional to the speed, appears, and hence the E.M.F. locus is again a straight line parallel to OI, as in the series motor. If the machine is driven in the opposite direction as a generator, we again reach a point OM where current and terminal voltage are exactly in quadrature.

Thus the E.M.F. locus for constant current is identical with that of the neutralized series motor, and we can derive the current locus, and the construction for determining the speed,

* F. Creedy. "The Alternating Current Series Motor." Journal I.E.E., 1905.

from the above diagram by exactly the same argument as that used in the series motor, and in fact the two diagrams are identically the same.

To obtain the secondary current, it will be convenient to proceed algebraically, as this seems to be the best way of discussing a divided circuit.

Consider the circuit passing through the field coil F, the neutralizing coil N, and the shunt coil S.

Let z be the impedance of the field and neutralizing coils, and z_0 that of the shunt coil.

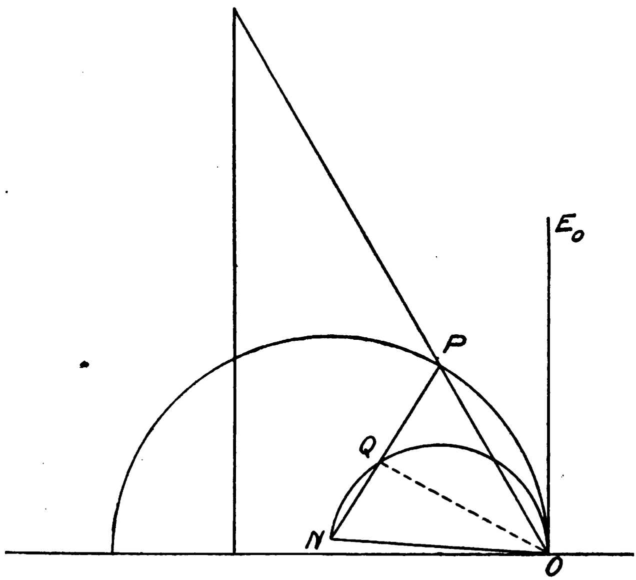


FIG. 25.

The current in F is i, that in S i_0, and that in A i_1.

Then $$i_0 + i_1 = i \text{ or } i_0 = i - i_1.$$

Equating the E.M.F.'s in the circuit including the shunt to zero

$$e_0 - iz - i_0 z_0 = 0.$$

Substituting, we get

$$e_0 - iz - (i - i_1)z_0 = 0,$$

or

$$e_0 - i(z_0 + z) + i_1 z_0 = 0.$$

Transposing,

$$i - \frac{e_0}{z_0 + z} = i_1 \frac{z_0}{z_0 + z}.$$

This equation asserts that the secondary current i_1, multiplied by a constant $\frac{z_0}{z_0 + z}$, is equal to the vector difference between i and the fixed current $\frac{e_0}{z_0 + z}$, which is the current which would flow if the rotor circuit were entirely open.

Referring to our diagram, the equation is interpreted as follows:

We saw above that the line current i, moved on a circle identical with that on which the current in the neutralized series motor moves.

In this diagram (Fig. 25) we may represent the current $\frac{e_0}{z_0 + z}$ by the vector ON. If OP represents the line current, then the above equation shows that NP will be proportional to the armature current.

Torque of the Repulsion Motor.

The armature current in the repulsion motor is no longer in phase with the flux as it is in the series motor, and hence the torque construction is different.

The torque is proportional to the mean product of flux by armature current. The flux is proportional to the primary current, hence the torque will be proportional to the product of the secondary current into the component of the primary along it, that is, to NP.PQ in the diagram, where PQ is the projection of OP on NP. Since the angle OQN is always a right angle, it follows that the point Q describes a semicircle on ON as OP varies. This fact gives us our most convenient means of locating Q, which may be defined as the intersection of NP and the semicircle.

It will be noted that the semicircle OQN intersects the current circle at a certain point. At this point the flux is in quadrature with the field current, and hence the torque is zero, and the machine cannot rise above the corresponding speed. This is a characteristic peculiarity of the repulsion motor, which is the only one of the machines discussed here which will not "run away" on no load.

The Inverted Repulsion Motor.

The inverted repulsion motor, as we saw above, is equivalent to a single-phase series motor with the shunt-coil connected across the neutralizing and field circuits in series.

In this machine the line current, after passing through the armature, divides into two, part passing through the shunt-coil and part through the neutralizing and field coils. As the impedances of these two paths are both constant and independent of the load, the current divides in invariable ratio between them, and hence the field current, though no longer equal to the line current nor even, necessarily, in phase with it, is accurately proportional to it. In fact, the shunt-coil S simply performs the function of an inductive shunt, such as is used in wattmeters or other measuring instruments.

The diagram, therefore, is identical with that of the series motor, with an appropriate change of scale to take account of the effect of the shunt, and so we need not stop to discuss it afresh.

The Induced Series Motor.

The induced series motor, as we saw, was equivalent to an ordinary series motor in which the coil S is joined in series with the neutralizing coil across the line. Practically, the sole effect of this is to cause the motor to take a constant magnetizing current in addition to its armature or load current.

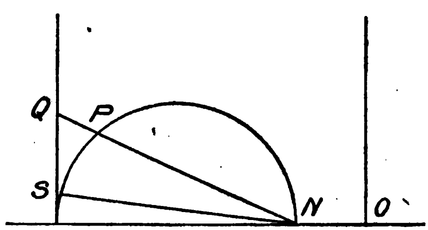


FIG. 26.

The diagram of this machine is identical with that of the series motor in all other respects, and all the arguments which we applied to the series motor are also applicable here. The presence of this magnetizing current, however, moves the circle away from the origin by a length ON equal to that of the magnetizing current vector. Hence the diagram takes the form

of Fig. 26, in which the circle, unlike that of all the other series type motors, does not pass through the origin. In this diagram OP represents the primary and NP the secondary current while SP is proportional to the speed as before.

The Compensated Repulsion Motor.

The compensated repulsion motor is, as we saw, identical with the ordinary repulsion motor, save that the coil which excites the motor field is now on the rotor instead of the stator. This modifies the performance diagram very materially.

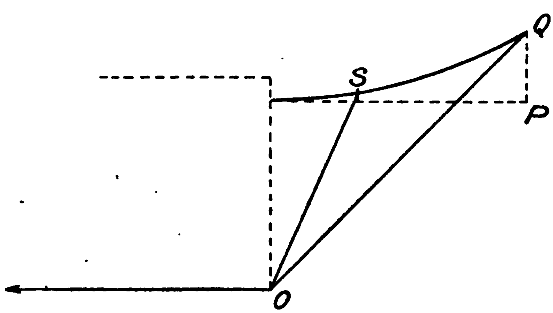


FIG. 27.

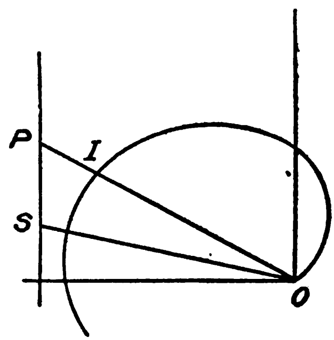


FIG. 28.

First, consider the constant current diagram. This will be the same as that of the repulsion motor, save that the flux due to the shunt-coil Z_0 now induces an E.M.F. of rotation in the field circuit XX.

At starting, then, we take OS (Fig. 27) as the E.M.F. corresponding to the constant current i, just as in the repulsion motor. The E.M.F. across YY is now almost zero, being only that corresponding to its resistance and inductance drops.

The flux in the shunt-coil is therefore very small.

As we speed up, the current remaining constant, an E.M.F. directly proportional to the speed and in phase with the current is induced in the YY axis just as in the repulsion motor. This we set off in our diagram as SP just as before.

The E.M.F. across the armature, however, excites a flux proportional to and in quadrature with itself in the shunt-coil Z_0 and this flux produces an E.M.F. in phase with itself, that is, in

quadrature with SP in the circuit through the XX brushes. Let us set this off as PQ.

PQ is proportional to k times SP, which is itself proportional to the speed. Hence PQ is proportional to the square of the speed.

The locus, therefore, of the E.M.F. vector as we vary the speed is now not a straight line but a curve of the second order—in fact, a parabola.

The projection of the point Q, the extremity of the E.M.F. vector, on a line SP parallel to the current vector, is still proportional to the speed, and when we reverse the motor we again reach a speed at which the E.M.F. vector OM is exactly in quadrature with OI.

Taking the reciprocals of all such vectors as OQ we get the locus of the current vector for constant E.M.F., which is now no longer a circle but a quartic curve (see Fig. 28) of the same general type as the cardioide. Since the parabola is a curve which goes to infinity at one point, it is clear that its reciprocal must pass through the origin.

Summary.

The brief sketch given above will serve to show how we may answer the question, What are the characteristics of this type of machine with varying speed?

A quite similar set of diagrams may be constructed for the shunt-type motors, and these have likewise been developed by a number of authors.* It is not proposed to discuss these here, as it will probably prove more profitable to devote our space to the second of the questions propounded in Chapter. I.

A list of references to articles on the single-phase motor will be found below.

LIST OF REFERENCES.

BOOKS.

Punga: "Single Phase Commutator Motors" (Whittaker).

Goldschmidt: "Alternating Current Commutator Motors" (*Electrician*).

Wilson and Lydall: "Electric Traction" (Vol. 2) (Arnold).

Arnold and La Cour: "Die Wechselstromtechnik"; Vol. 5: "Die

* F. Creedy. "The Shunt Induction Motor." Trans. Amer. I. E. E., 1909.

Asynchronen Wechselstrommaschinen"; Part 2: "Die Wechselstrom Kommutatormaschinen" (Julius Springer).

Dyhr: "Der Emphasen Motoren" (Julius Springer).

ARTICLES.

An enormous number of articles have appeared on the subject since 1902, chiefly in the Continental electrotechnical journals, such as the *Electrotechnische Zeitschrift* and *L'Eclairage Electrique.* It would be impossible to give references to each of these but some of the articles by the following prominent workers are of special interest:—

Blondel, Bethenod, Latour, Osnos, Bragstad, Fraenckel, La Cour, Eichberg, Schnitzler, Richter, Behn-Eschenberg.

Practically the whole of these articles and books deal either with the vector-diagrams, on lines more or less similar to the brief *résumé* of the present chapter, or with the design, but many of the articles on "design" give analyses of existing designs, in which the quantities to be determined by the designer are given in advance, rather than information as to how these quantities may be determined. M. Bethenod was perhaps the earliest to give a full theoretical treatment of the shunt type motors.

CHAPTER III

FLUX AND CURRENT DISTRIBUTION IN ALTERNATING CURRENT MOTORS

We shall now endeavour to investigate the flux distribution within such an alternating current motor. It has been known for a long time that in general the field distribution of an alternating current motor was elliptical in form, if we assume that the windings are harmonically distributed. The simple alternating field, such as is found in the single-phase series motor, and the purely rotating field, such as we find in the induction motor, are simply special cases of the elliptical field.

It has been believed hitherto, however, that it was impossible to treat of an elliptically distributed field directly, but that before it could be advantageously dealt with it must be analysed either into two oppositely rotating fields or into two component fields along axes fixed in space.

This belief, however, is entirely without foundation, and the result of resolving an elliptical field into components is simply to build up an artificial scaffolding round the actual occurrences, through which it is often quite difficult to catch even a glimpse of what really goes on.

In the present investigation, wherever we come across an elliptical distribution of current, electromotive force, or magnetic flux, we shall treat it as such without any attempt to reduce it to so-called simpler components.

Fundamental Assumptions.

In order to assist us in our investigations of first principles, the following fundamental assumptions will be made and rigidly adhered to :—

(1) All periodically varying quantities will be assumed to vary in strict accordance with a sine law in time.

(2) All windings will be assumed to be harmonically distributed around the periphery of the machine.

The effect of departures from these assumptions may be investigated later.

Before we can endeavour to discuss any specific type of machine,

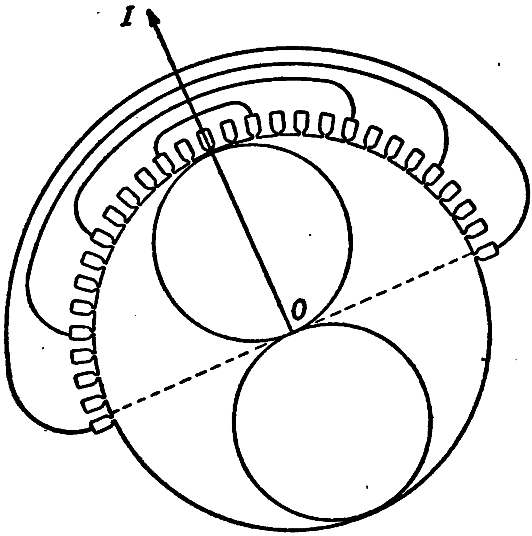


FIG. 29.

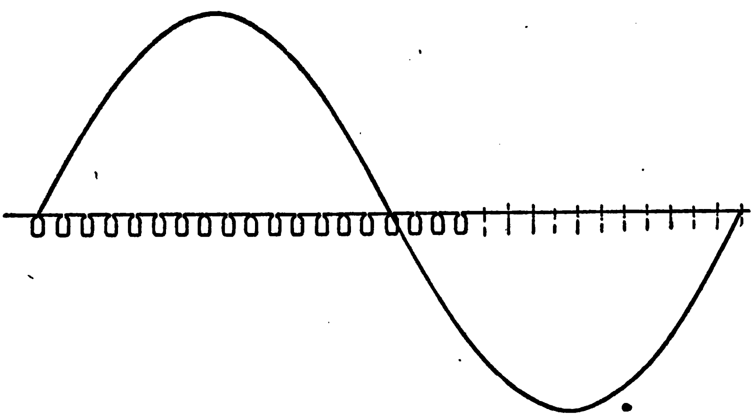


FIG. 30.

it will be necessary for us to investigate the nature and properties of the elliptical field and also to know a little of the geometry of the ellipse.

The treatment which we shall adopt will be almost exclusively

graphical. We shall find it necessary, in the first place, to abandon the ordinary phase diagram, in which the angle between two vectors represents the phase difference between them. In the diagram which we shall draw the angle between two vectors will represent the angle between them in *space*, not in phase.

Our investigations deal with electrical machinery, composed of a slotted stator with uniform air-gap all round and a rotor of a similar type. Upon the stator are any number of windings variously connected, and the same upon the rotor.

It is assumed, as stated above, that all these windings are harmonically distributed. For instance, if Fig. 29 represents such a stator bearing a certain harmonically distributed winding and Fig. 30 represents the same developed, then a current flowing through such a winding produces a magnetomotive force which is represented in rectangular co-ordinates in Fig. 30, and in polar co-ordinates in Fig. 29. Clearly, such a magnetomotive force may be completely represented by a vector whose direction is that of the axis of the coil and whose magnitude represents, to some convenient scale, the total magnetomotive force due to the coil.

For, given such a vector, we should know how to wind our coil so as to produce the required distribution of magnetomotive force, and when it was wound with any desired number of turns what current to put through it to get the desired intensity.

It should be particularly noted that this vector represents a *constant* magnetomotive force and therefore has a radically different meaning from the vector of the phase diagram which is quite incapable of representing a constant quantity.

If such a vector should be variable in magnitude we represent its variation by varying the length.

If it should be harmonically variable, the length of the vector may be taken to represent its maximum value. If, then, we can represent its phase also, we can clearly make our vector a complete representative of such a magnetomotive force. This can easily be done by drawing our vector having a certain direction—that of the axis of the coil to which it is due—a certain length representing the maximum magnetomotive force in the coil and making a mark on it, say, an arrow-head, showing the magnitude of the quantity at some particular time, say, time O, and also whether it is rising or falling. Such a vector is represented at

OA in Fig. 31. From this information we may very easily derive the value of the quantity at any other time.

Draw a circle of radius OA and centre O. Project the point B, indicated by the arrow-head, upon the circle and draw the radius OC, such that its projection on OA is OB. The direction of the arrow-head shows that OB is falling. Hence the direction of rotation of OC is as shown, and its speed of rotation may be calculated from the frequency, supposed to be known. At any given instant, then, the projection of OC on OA gives the value of the magnetomotive force. We may, in fact, regard the vector OA, with its phase-arrow, as we may call it, as an edge view of the circle which we draw when we wish to find the phase angle.

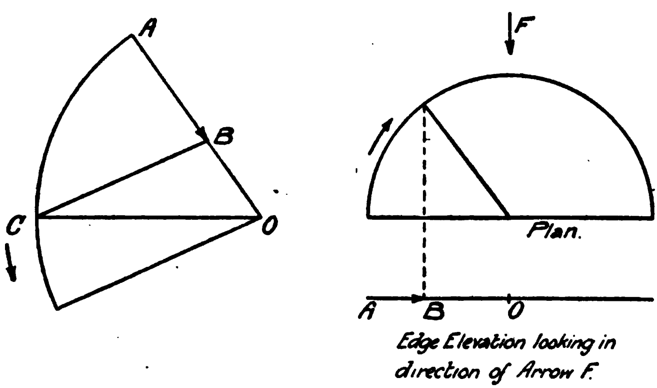


FIG. 31.

The plane in which the vector OA is drawn is that perpendicular to the shaft of the machine under study. We may regard the circle of Fig. 31 as being perpendicular to the plane in which OA lies and, therefore, as being in one of the planes passing through the shaft. Looking in the direction of the shaft or arrow F (Fig. 31), we shall see a mere straight line, the vector OA, an edge view of our circle.

We shall find this conception occasionally useful, though not essential, as we proceed.

We may note that such a vector as OA, with its phase-arrow, requires three independent quantities to specify it completely: (1) the direction of OA; (2) the length of OA; and (3) the length of OB, or equivalent information. Whereas, of course, the ordinary vector of the phase diagram requires but two.

At any given instant such vectors may be compounded by the regular rules of vector addition, as may easily be seen if we consider what they represent. Each of two such vectors, as OA and OB (Fig. 32), represents a harmonically distributed magnetomotive force, shown developed on the right.

Adding the sine wave A to the sine wave B at every point of the periphery, we get the sine wave C, which, as may easily be

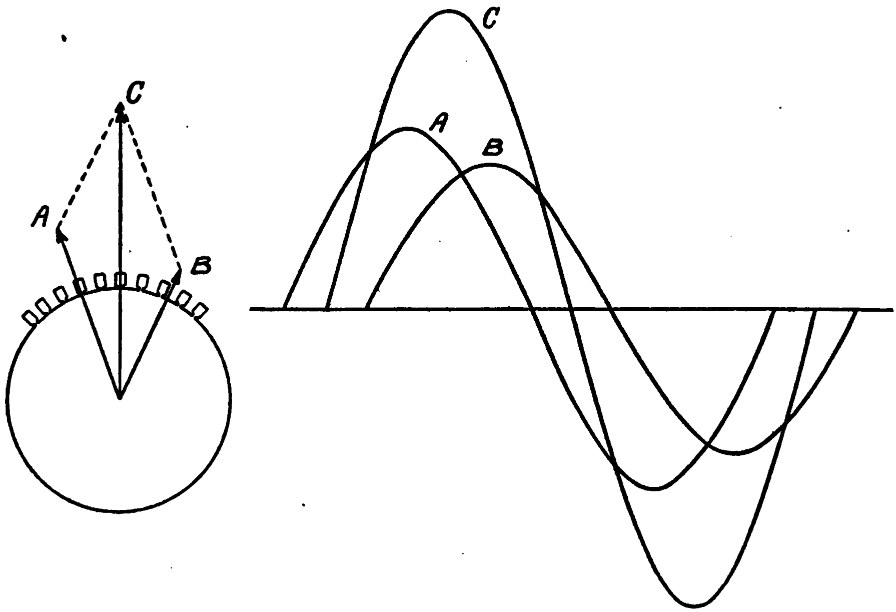


FIG. 32.

proved, may be represented by the vector OC in magnitude and position. Thus, the ordinary rule of vector addition serves us to find the resultant of two constant magnetomotive forces or of two magnetomotive forces at any given instant.

To find the complete resultant of two such variable vectors, we must take their resultants at every instant by vector addition. The resultant vector will then describe some curve which we shall call the resultant of the two variable vectors. The result of this process is shown in Fig. 33, in the case of two harmonically oscillating vectors which are in the case given supposed to be in quadrature. Adding the corresponding values of the two

vectors at a number of successive instants, we get the resultant curve, an ellipse. In the diagram, corresponding values are numbered the same. It is very easy to prove that the resultant of any number of harmonically oscillating vectors is an ellipse. For each vector, by a well-known theorem, may be resolved into two oppositely rotating vectors, which are such that at any instant their resultant is equal to the value of the oscillating vector at that instant (see Fig. 34).

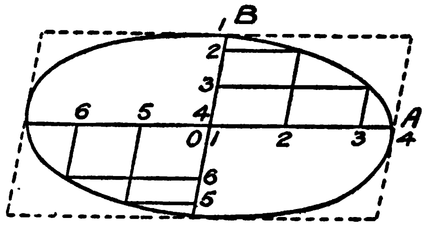


FIG. 33.

Take any two oscillating vectors OA and OB (Fig. 35) and resolve each of them into two oppositely-rotating components OA′, OA″, and OB′, OB″. Add together, vectorially, those rotating counter-clockwise, viz., OA′ and OB′, and we get as resultant OC′. Adding OB″ and OA″, which rotate clockwise, we get OC″. Now

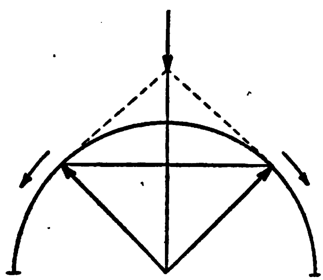


FIG. 34.

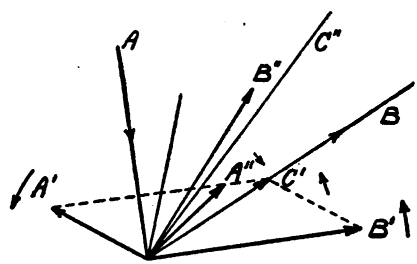


FIG. 35.

if we add OC″ and OC′ at each instant, we get an ellipse whose major semi-axis is equal to the arithmetical sum of OC″ and OC′ and whose minor semi-axis is their difference.

Two oppositely rotating vectors of unequal length always give rise to an ellipse as resultant, while, if they are equal, the ellipse degenerates to a straight line.

Hence, since the various windings on our stator and rotor all produce such oscillating vectors, we may take it for granted that our investigations will deal with such ellipses as we have just deduced. It is therefore necessary to investigate their properties.

Such an ellipse requires four independent quantities to specify

it, namely, the major and the minor axis, the angle a, Fig. 36, and some quantity specifying the position of the radius vector at time O or at some given instant. This may be the angle β between the radius vector at time O, and the major axis or any quantity giving equivalent information.

For instance, the projection of the radius vector on the major axis would give the same information as that given by the angle β.

Such an ellipse may be described in a clockwise or a counter-clockwise sense. Hence we use an arrow-head on the circumference to denote at once the position of the radius vector at time O, and the direction of rotation.

If the minor axis of our ellipse vanishes, the ellipse degenerates to an oscillating vector, such as we have treated of above, requiring only three quantities for its specification.

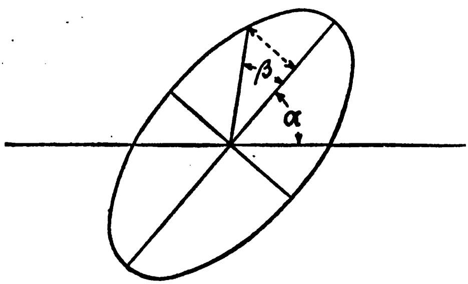


FIG. 36.

Hitherto we have spoken as if the vectors, ellipses, etc., under discussion must always represent magnetomotive forces. This is by no means the case, as they are equally well adapted to represent fluxes, currents, and E.M.F.'s. It will be convenient here to adopt the following definitions. In an harmonically distributed coil in which an alternating E.M.F. or current occurs, it will be represented by a vector parallel to the axis of the coil, and of a length corresponding, to any given scale, with the maximum value of the E.M.F. or current. The phase will be represented by a phase-arrow in the manner described above. An harmonically distributed flux will be represented by a vector directed from the centre of the machine to the point of maximum density, and of a magnitude such as to represent the maximum value of the flux to a convenient scale, while its phase is represented by a phase-arrow, as indicated above.

In purely geometrical arguments, where the quantities used may equally well represent currents, E.M.F.'s, or anything, we shall speak simply of vectors and vector ellipses.

The same method of proof is equally applicable no matter how many oscillating vectors we have. We can resolve each of them into two oppositely rotating vectors, add all those rotating clockwise and all those rotating counter-clockwise, and then compound the two resultant vectors at each instant, when we get, of course, an ellipse. Hence we may say, generally, the distribution in time and space of the harmonically variable quantities in an electric machine is in accordance with such an ellipse as has been described above. This ellipse may occasionally degenerate to a straight line or a circle.

By the fundamental definition of the meaning of the vectors used in this investigation, each represents a distribution of, say, magnetomotive force, current, E.M.F., etc., around the periphery

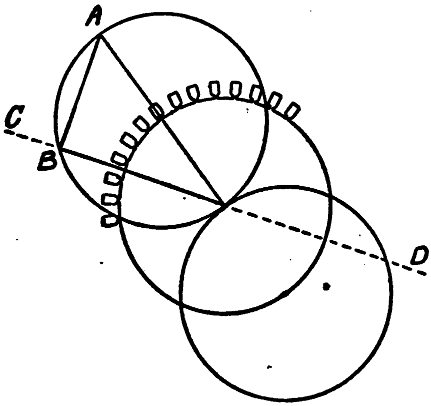


FIG. 37.

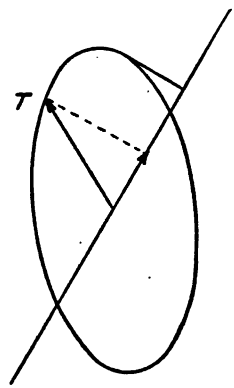


FIG. 38.

of a machine, which would be represented in polar co-ordinates by a pair of circles (see Fig. 37). The magnetomotive force along any axis CD is given by the intercept OB of the circle on that axis, or, since the angle OBA is a right angle, we may say that the magnetomotive force along any axis OD due to any coil whose actual magnetomotive force is represented by the vector OA is equal to the projection OB of the vector OA on the given axis CD.

This also holds if OA is a variable vector, and hence it is true of the successive radii vectors of an ellipse. Hence we may say that any given vector ellipse gives rise, along any given axis, to an oscillating vector which is the projection of the ellipse upon the given axis.

The maximum value of the oscillating vector is found by drawing a tangent to the ellipse perpendicular to the given axis, and the position of the phase-arrow is found by the projection of the phase arrow of the ellipse (see Fig. 38).

It was remarked above that an oscillating vector might be regarded as the edge view of a circle in the plane of which circle the ordinary vector of the phase diagram rotates. Similarly, an ellipse may be regarded as the plan view of such a circle inclined to the paper by an angle whose cosine is the ratio of the minor to the major axis.

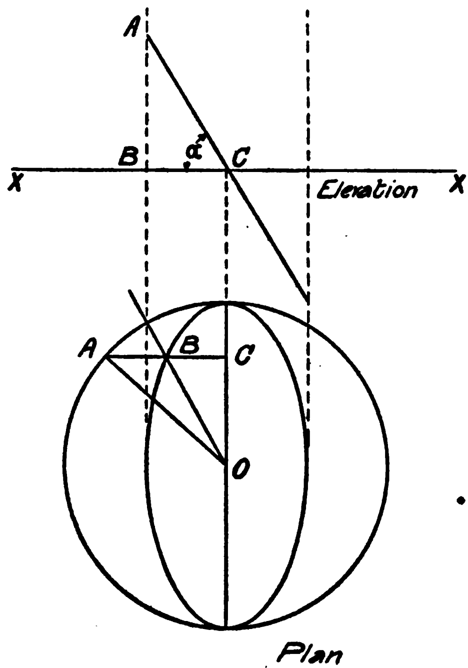


FIG. 39.

Suppose we take any given ellipse and draw a circle touching it at the two extremities of the major axis. Suppose this ellipse to be drawn in the horizontal plane and that the circle at first lies in the same plane. Suppose, next, that the circle is tilted in the manner shown in the elevation given in the upper part of Fig. 39. If, then, we continue to project the tilted circle on the horizontal plane, in the plan of this circle all lines such as AC, perpendicular to the axis round which the circle is turned, will be shortened in proportion to the cosine $\frac{BC}{AC}$ of the angle α, while those parallel to the axis OC of tilting will be unchanged. Hence, when the circle has been tilted to the degree shown, OA will be projected into OB, the corresponding radius vector of the ellipse.

Another method of regarding the same question is as follows: Every ellipse may be regarded as compounded of two oscillat-

ing vectors in quadrature and at right angles. These are shown in Fig. 40 at OA and OB, their phase-arrows being G and E. Drawing the same circle as before, it is clear that the projection of OC, as well as of OH, on OA is OG. From the known fact that the projection of a circle tilted in the manner described above is an ellipse, and since but one ellipse can be drawn with major axis OA and minor axis OB, it follows that $\frac{GH}{GC}=\frac{OB}{OD}$. For, when the circle is tilted round the axis OA, all lines parallel to OD are shortened in the same proportion. Hence, if OD is shortened to OB, then OC is shortened to OH. Hence we have

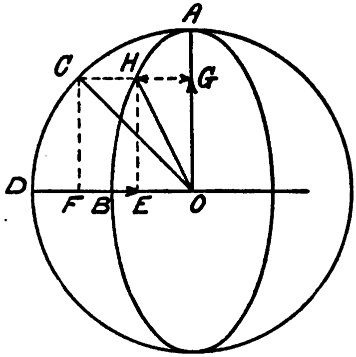


FIG. 40.

$$\frac{GH}{GC}=\frac{OE}{OF}=\frac{OB}{OD}.$$

Hence, transposing,

$$\frac{OE}{OB}=\frac{OF}{OD}.$$

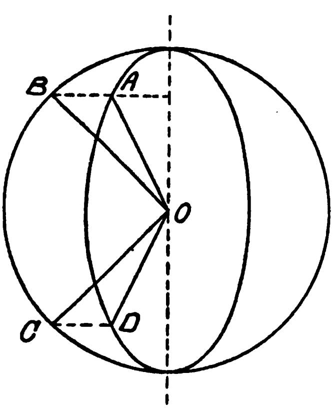


FIG. 41.

Hence the circle is capable of giving us the phase of OB as well as that of OA.*

Now the radius OC of the circle rotates uniformly and, at any instant, the projection of this radius on the ellipse, perpendicular to the major axis, gives the radius vector of the ellipse at that instant.

If we have the position of this radius vector given at some particular instant, say time O, then, to find its value for any other time the procedure will be as follows:

Draw a circle touching the ellipse at the two extremities of the major axis. Project the phase-arrow of the ellipse A (Fig. 41)

* These propositions, though interesting, are not of primary importance so the student need not spend too much time on them.

on the circle by a line perpendicular to the major axis thus giving the radius OB. Suppose we wish to find the radius vector of the ellipse a quarter period later, which is represented by 90° on the diagram. Draw a radius OC to the circle, making 90° with OB in the proper direction. Project this on the ellipse perpendicular to the major axis thus obtaining the radius vector OD. This represents the value of the radius vector a quarter period after the moment represented by OA. The angle between OA and OD will not be 90° and, therefore, it follows that this radius vector does not rotate uniformly. Although it is not worth while to stop to prove it, it may be of interest to remark that it follows Kepler's law and covers equal areas in equal times.

To recapitulate for a moment. Such an ellipse as we have been discussing represents completely the distribution of any harmonically varying quantity round the periphery of an electric machine in magnitude, position, and phase. In the case of the ellipse, the representation of the phase by a phase-arrow is nearly as graphic as the representation in an ordinary phase diagram, which, however, does not represent the position in space of such quantities at all. The ellipse is specified by four quantities, and it should be carefully noted that the drawing of such an ellipse does not completely specify it unless the phase-arrow is marked on it as well.

From the ellipse we can deduce immediately the oscillating vector representing the values of the quantity along any given axis. But it would be a great mistake to resolve our ellipse into components along two such axes. This is what has hitherto been done, but with very unsatisfactory results. The ellipse is manifestly a single whole, and should be treated as such. Having decided to endeavour to treat the ellipse as a whole, it becomes necessary to obtain some knowledge of its geometry. We shall only require a few propositions, and these will be stated in tabular form, as clearly and concisely as possible. Proof will not, as a rule, be attempted, but a reference will be given in most cases to a treatise where a proof may be found. (See Appendix.)

Conjugate Diameters.

Draw any diameter of an ellipse and a tangent at the extremity thereof. Draw a second diameter parallel to this tangent. This

diameter is said to be conjugate to the first. It may be proved that if any diameter BB (Fig. 42) is parallel to a tangent drawn at the extremity of another diameter AA, then the diameter AA is parallel to the tangent at the extremity of BB. Hence—

Two conjugate diameters are two diameters such that each is parallel to the tangent at the extremity of the other.

We shall make constant use of the above definition.

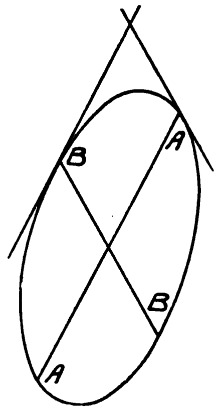


FIG. 42.

Note also that tangents at opposite extremities of a diameter are parallel to one another. This may easily be seen by considering the ellipse as the projection of a circle.

In a vector ellipse with rotating radius vector, it may easily be proved that conjugate diameters are in quadrature with one another.

Let us take two oscillating vectors in quadrature with one another and making any angle in space and consider the ellipse which they generate.

An examination of Fig. 43 will show that at time 4, when OA is zero, the vector OB is a maximum and the ellipse, the resultant of the two, reaches the line BC. At any other time the vector OB has a smaller value than at time 4, and hence the ellipse can only reach the line BC at that particular time. Hence the ellipse is tangent to the line BC at B. Similarly, we may prove that it is tangent to AC at A. Now BC is parallel to OA and AC to OB. Hence OA and OB are conjugate diameters.

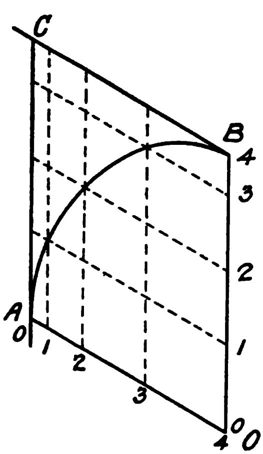


FIG. 43.

The same proposition may easily be proved by considering the ellipse as the projection of a circle.

In the circle, two diameters which are in quadrature will be at right angles to one another, as at OB, OA, Fig. 44.

Now AC, the tangent at A, is at right angles to OA, and BC is at right angles to OB. Therefore AC is parallel to OB and BC

to OA. Now, on projection, parallel lines remain parallel lines, and hence when we project the circle into an ellipse AC remains parallel to OB and BC to OA.

Hence we may conclude—

The conjugate diameters of a vector ellipse are in quadrature with one another in time.

This will be one of our most important propositions.

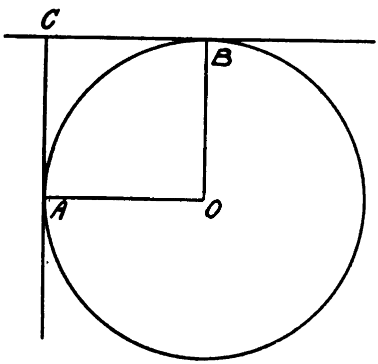


Fig. 44.

This is all that is really necessary of the geometry, but in the Appendix are some reference propositions.

When applying the above methods to actual machines we very frequently find that it is required to draw an ellipse when two conjugate diameters are known. One of the simplest ways of doing this is described below. Any vector ellipse, as we saw above, may be resolved into two oppositely rotating vectors. To find the two rotating vectors corresponding to two given conjugate axes, proceed as follows :

Let OA, OB be the two conjugate axes. Draw vectors OA″

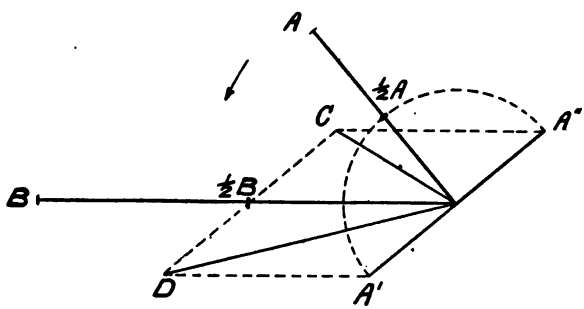


Fig. 45.

and OA′ leading and lagging 90°, respectively, on OA, which, of course, may be either conjugate diameter.

Then $\quad 1/2\,\mathrm{OB} + \mathrm{OA}' = \mathrm{OD}$

and $\quad 1/2\,\mathrm{OB} + \mathrm{OA}'' = \mathrm{OC}$

are the two rotating vectors required.

To find the major and minor axes of the ellipse, given the two oppositely rotating vectors of which it is composed, bisect the angle between them by the line EE. Draw FF at right angles to EE (see Fig. 46).

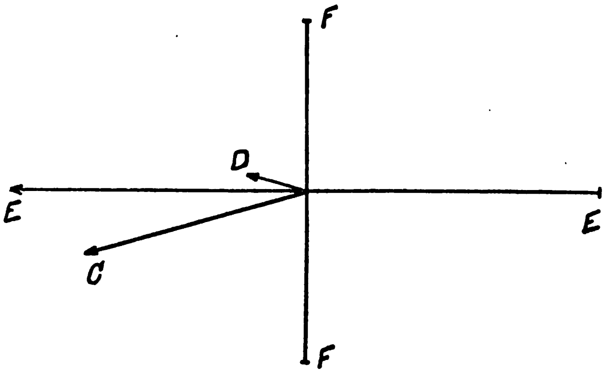


FIG. 46.

Set off their arithmetical sum at OE along the line bisecting the angle between them and their arithmetical difference at OF along the line at right angles thereto. OE is now the major and OF the minor axis of our ellipse which can thus be drawn in by the trammel method or otherwise.

CHAPTER IV

FLUX AND CURRENT DISTRIBUTION IN ALTERNATING CURRENT MOTORS (*continued*)

HAVING obtained clear ideas as to the properties and geometry of our elliptical quantities, we have next to apply our results to actual electromagnetic machines. Let us consider, in the first place, a machine with drum-wound stator and drum-wound rotor.

Let there be a certain elliptical field crossing the air-gap and interlinking both stator and rotor.

We shall consider, for the moment, that both stator and rotor are at rest and contain the same number of turns. Then the assumed elliptical field will induce in both a certain ellipse of electromotive force which will be similar in shape and position to the flux ellipse.

This may easily be seen if we resolve the flux ellipse along its major and minor axes into two oscillating or single-phase fluxes at right angles in space and in quadrature in time. Then each such flux produces an E.M.F. along its own axis *proportional to itself*, and hence the ellipse compounded of the two E.M.F's along these two axes will be similar and similarly situated to the original flux ellipse.

In general, we can find the flux interlinking a circuit having any axis by projecting the flux ellipse on that axis, and the E.M.F. in such a circuit will always be proportional to the flux.

Now this E.M.F., although proportional to the flux, is in quadrature therewith so that the phase-arrow on the E.M.F. ellipse will be in quadrature with that on the flux ellipse.

The numerical relation between the two ellipses is the familiar one relating the *maximum* values of any alternating flux with the E.M.F. it produces, *i.e.*,

$e = 2\,\pi fqn\,\phi\,10^{-8}$,

$q =$ breadth coefficient

$f =$ frequency,

n = number of turns in series between points electrically 180° apart on the drum winding,

ϕ = flux.

A certain magnetizing current must also flow in order to supply this flux ellipse. It will be clear that the ellipse of magnetizing current must also be similar and similarly situated to that of flux. They will, moreover, be in phase with one another neglecting the effect of hysteresis.

The numerical relation between them is again the familiar one (cm. units)

$$\text{amp.-turns} = 0{\cdot}8 \times \text{max. density} \times \text{length gap} \times S,$$

S being the "saturation factor" taking account of the reluctance of the iron circuit.*

Hence the same ellipse may represent, to different scales, three different quantities—the flux, the magnetizing current, or the E.M.F. (in the latter case requiring a different position of the phase-arrow). The relation between the different scales is given by the formulæ quoted above.

Besides the air-gap flux, discussed above, there will, in general, be two other fluxes—the stator leakage flux, which is proportional to the stator current, and the rotor leakage flux, which is proportional to the rotor current.

It is impossible to discuss the distribution of these fluxes in general, or, indeed, the distribution of the stator and rotor currents, which comes to the same thing, until the connections of the machine are specified.

Now, although the stator and rotor leakage fluxes may be regarded as distinct, physically, from the air-gap flux, yet they induce E.M.F's in the same circuits as the latter does, which E.M.F's may be compounded into a single one with it. It is therefore convenient to compound them with the air-gap flux and establish the following definitions:

The "total stator flux" is the resultant of the air-gap flux and the stator leakage flux.

The "total rotor flux" is the resultant of the air-gap flux and the rotor leakage flux.

We can now say that the "stator induced E.M.F." is the E.M.F. induced by the total stator flux in the stator winding,

---

* S may vary along different axes which will distort the ellipse somewhat.

and that the "rotor induced E.M.F." is that induced in the rotor winding by the total rotor flux.

We have next to inquire what takes place when a flux ellipse cuts a revolving rotor, for hitherto we have only discussed the case where both rotor and stator were stationary. Before doing so we shall find it convenient to introduce the idea of the "complement" of any vector ellipse.

Definition.

The complement of any vector ellipse is another ellipse such that the sum of the given ellipse and its complement is a purely rotating vector whose magnitude is the arithmetical sum of the axes of the ellipse.

For instance, the complement of a single-phase oscillating vector in the case where the ellipse degenerates to a straight line is another similar oscillating vector at right angles to it in space and in quadrature with it in time. Thus, if YY (Fig. 47) be the original oscillating vector which is at its maximum at the instant shown by the phase-arrow, its complement will be XX, which is zero at the same instant or in quadrature with YY, and also at right angles to it in space. The resultant of these two is clearly a simple revolving vector rotating counter-clockwise.

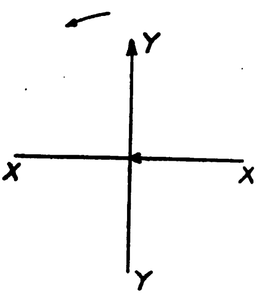


FIG. 47.

Any ellipse may be resolved into two mutually perpendicular oscillating vectors along the major and minor axes.

The complement of the major axis of the ellipse will be the major axis of the complementary ellipse, and the complement of the minor axis will be the minor axis of the complement. Hence, the complementary ellipse as a whole will be exactly similar to the original ellipse but turned through an angle of 90° in space and with its phase-arrow displaced a quarter-period in the manner shown in the figure.

Thus another definition of the complementary ellipse would be as follows:

The complement of any ellipse is another equal ellipse at right angles to the first in space, rotating in the same sense and with

radius vector having the same direction when passing the axes (see Fig. 48).

Consider, now, the case of a simple alternating or single-phase flux interlinking a revolving rotor which we will suppose is fitted with a commutator and brushes. Across the *yy* brushes (Fig. 49)

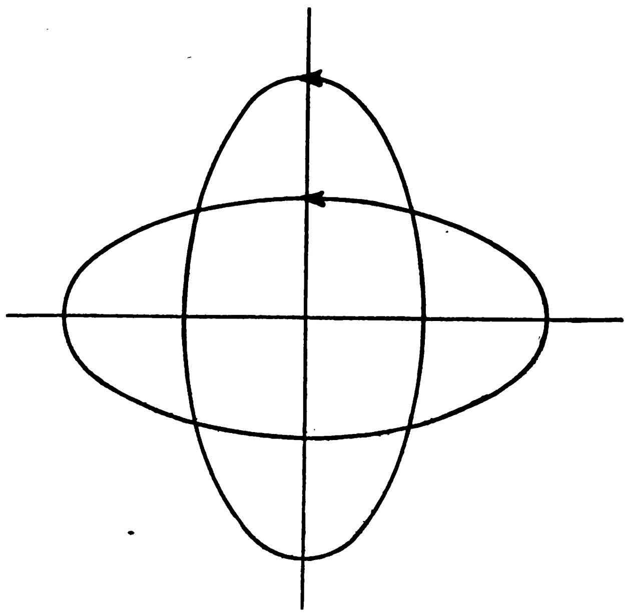


Fig. 48.

whose axis is parallel to the axis of the flux, an E.M.F. will be generated just as in a transformer, due to the alternation of the flux and proportional to the frequency. This may be called the "transformer E.M.F." and is proportional to the rate of change of the flux and hence would not occur if the latter were constant. Where we have a sine wave of flux, as we assume throughout, this E.M.F. will lag 90° behind the flux.

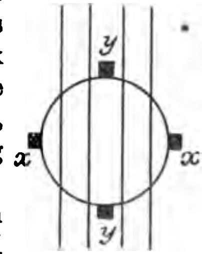


Fig. 49.

When the rotor begins to revolve, an E.M.F. will appear across the brushes XX which is proportional to the speed and independent of the frequency. This E.M.F. may be called the "E.M.F. of rotation" and would be exactly the same in magni-

tude whatever the frequency of the flux or even if it were constant. It is, in fact, the same E.M.F. as occurs in a direct current dynamo. It is in phase with the flux producing it. Thus a single-phase flux produces in a revolving rotor two E.M.F.'s both proportional to itself. One of these lies along the same axis in space as the flux, but lags 90° behind it in time. The other is in phase with it in time but makes an angle of 90° with it in space.

These two E.M.F's will be equal at synchronous speed, which may be defined as a speed such that a point on the rotor travels over twice the polar pitch (or makes a complete revo-

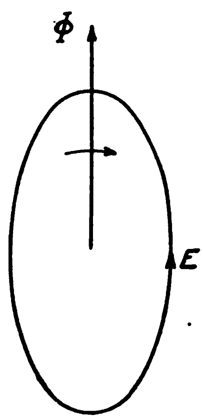


FIG. 50.

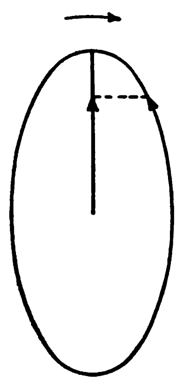


FIG. 51.

lution in the bi-polar machine) in the periodic time of the alternating current.

At this speed the rotation E.M.F. is the "complement" of the transformer E.M.F., and since at any other speed the rotation E.M.F. is k times as great as at synchronism, where k is the ratio of the speed to synchronism, the definition above enables us to state concisely:

The E.M.F. induced in a rotor revolving at speed k times synchronism by a single-phase flux is equal at any speed to the standstill E.M.F. plus or minus k times its complement.

Geometrically these facts appear as follows:

A single-phase flux gives rise in a revolving rotor to an E.M.F. whose major axis (below synchronism) is parallel to

that of the flux, the ratio of whose semi-axes is the same as that of the speed to synchronism, and which lags 90° behind the flux (see Fig. 50).

The direction of rotation of the phase-arrow is *opposite* to that of the rotor.

We may modify this in the case where the E.M.F. at standstill is given.

If any flux gives rise to a single-phase E.M.F. in a rotor at standstill, then, at any other speed, k, it will give rise to an ellipse of E.M.F. whose minor axis (below synchronism) is parallel to that of the standstill E.M.F., the ratio of whose semi-axes is k, the same as that of the speed to synchronism and which rotates in the *opposite* direction to the rotor. The position of the phase-arrow will correspond with that on the standstill E.M.F. ellipse (see Fig. 51).

It is quite clear that the same rule which we obtained for the E.M.F. due to a single-phase flux is also applicable to any elliptic flux without further modification, since every elliptic flux may be resolved into two single-phase fluxes.

Hence, we may state that:

The E.M.F. induced in a rotor revolving at speed k times synchronism by any elliptic flux is equal to the standstill E.M.F. plus or minus k times its complement.

The minus sign is to be taken when the rotor revolves *with* the flux. This is easily remembered, as the E.M.F. is naturally less in this case.

If the semi-axes of the standstill E.M.F. ellipse are a, b, proportional to those of the flux ellipse, the corresponding axes of the resultant ellipse will clearly be $a \pm k\,b$ and $b \pm k\,a$.

As it is very important to obtain the utmost clearness on this point, it will be convenient to consider a little more closely how such an E.M.F. ellipse varies as we change the speed. The only case that presents anything remarkable is that where the rotor goes the same way as the radius of the ellipse. In the diagrams (Fig. 52 and the table on p. 45) are given illustrations of the changes which the resultant E.M.F. ellipse goes through at various speeds.

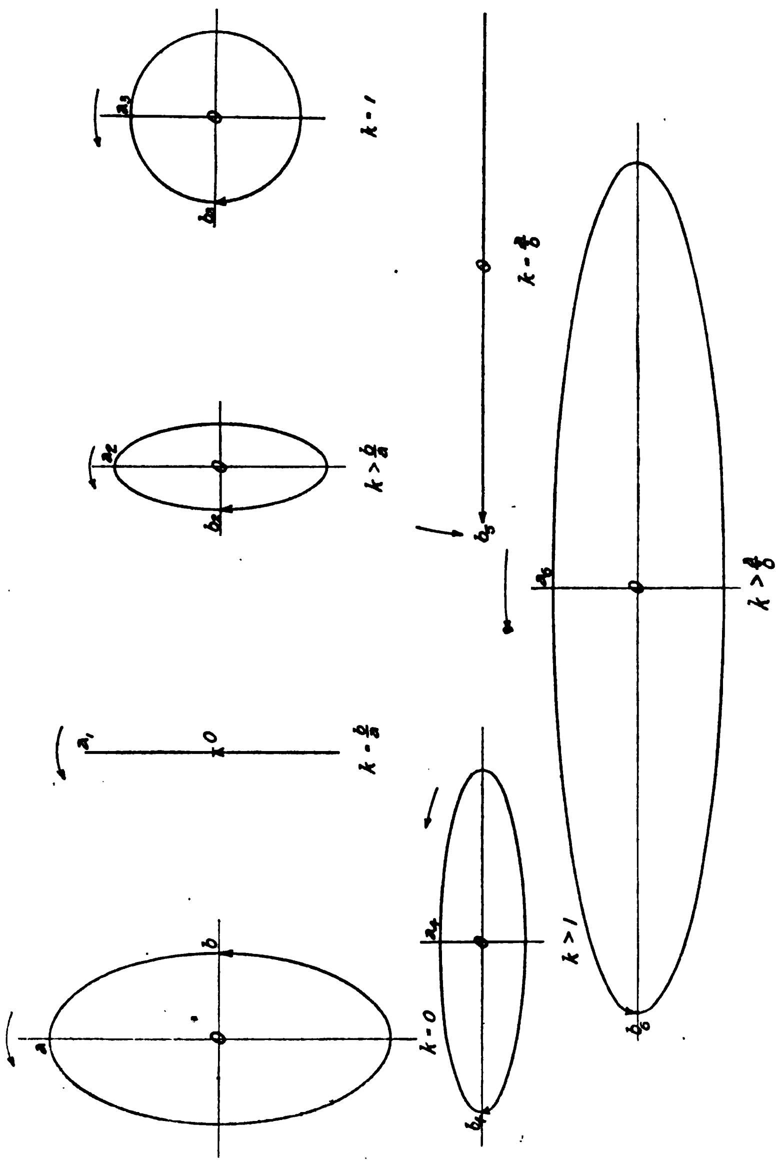


FIG. 52.

k	Vertical Axis.	Horizontal Axis.
0	a	b
$\frac{b}{a}$ *	$a - \frac{b^2}{a}$	0
1	$a - b$	$b - a$
$\frac{a}{b}$	0	$b - \frac{a^2}{b}$
∞	∞	∞

We see that first the ellipse collapses to a straight line along the major axis, then expands again, having reversed its direction of rotation till at synchronism it is a circle rotating in the opposite direction to the rotor. It then becomes elongated in the direction of the former minor axis, and finally collapses to a straight line again; after which it expands again, having again reversed its direction of rotation, and goes on expanding till it again becomes a circle at infinite speed.

If $a = b$, *i.e.*, the ellipse is a circle, the two speeds at which the ellipse collapses to a straight line, viz., $k = \frac{b}{a}$ and $k = \frac{a}{b}$, coincide at a value of $k = 1$. Hence, this double reversal of rotation does not take place at all in this case.

Before we go on to discuss the different types of alternating-current motor in detail, it will be convenient to say something about different methods of connecting the circuits of such machines.

The circuits may be connected in series or parallel, stator circuits with stator circuits, rotor circuits with rotor circuits, or stator circuits with rotor circuits. If we have two stator circuits at any angle connected in series, they may be compounded into a single circuit, since both are by hypothesis distributed harmonically. Circuits in parallel must be considered as independent whether on the rotor or stator.

* Conversely, if it is necessary that the horizontal axis, say, of the ellipse shall vanish at speed k, we must have

$$\frac{\text{Horizontal axis of flux ellipse}}{\text{Vertical axis of the flux ellipse}} = k.$$

Thus there only remains a stator circuit connected in series with a rotor circuit. These obviously cannot be compounded, as entirely different E.M.F's are induced in them by the same flux.

Let us inquire by what construction we may find the resultant E.M.F. in such a combined circuit due to given E.M.F's in each part. Let the two circuits have the axes AA′ and BB′ and the E.M.F's in them be OA and OB (see Fig. 53). In order to find the resultant E.M.F. vector we have to add the two variable E.M.F's OA and OB at every instant. We clearly cannot do

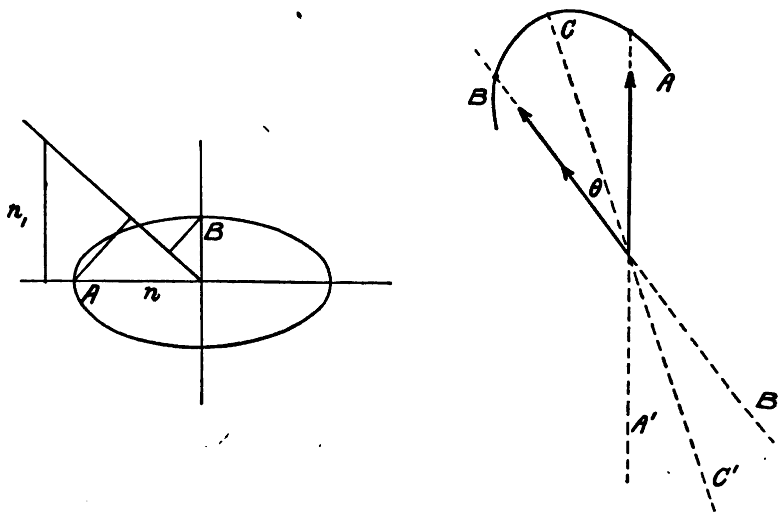


Fig. 53.

this directly, as they are not along the same line. Let us, then, draw a line CC′ bisecting the angle AOB. Let it make the angle θ with both OA and OB. Then, the projection of OA along it will be OA cos θ, and that of OB will be OB cos θ at every instant.

Hence, if we project both vectors along this line and add at every instant, then the resultant vector so obtained will be equal to (OA + OB) cos θ.

Now, if instead of projecting the two vectors on CC′ and then adding arithmetically at each instant, we had formed the vector sums of the original vectors, we should have obtained a certain ellipse as shown. And, if we had projected the radius vector of

this ellipse on CC′, we should have got exactly the same result as by projecting OA and OB separately and then adding.

Hence we get our final proposition:

If two E.M.F's exist in two circuits, making an angle with one another and connected in series, then the total E.M.F. across the two series circuits is found by adding these two E.M.F.'s at every instant to form a certain ellipse, projecting this ellipse on a line bisecting the angle between the two circuits and dividing the result by cos. θ where θ is half the angle between the circuits.

This is true whether the two circuits have the same number of turns or not.

A most important particular case of this proposition is that in which the angle between the two circuits is 90°.

Let us consider this case and generalise it to take account of the case when the two circuits have a different number of turns. By taking this explicitly into account we shall find later that our results are very much more useful, as our ellipses of current, E.M.F., etc., may be made to correspond.

Suppose for the sake of argument we have a circuit composed of two coils at right angles having axes OA and OB (see Fig. 53 on left) and having numbers of turns n and n_1 cut by a flux ellipse also having axes OA and OB, the ratio OB/OA being k.

$$E_1 = n\,\text{OA}$$
$$E_2 = n_1\,\text{OB}.$$

The E.M.F. across the terminals will be $E_1 + E_2$.

If we draw a line making such an angle θ with OA that $\tan\theta = \frac{n_1}{n}$, and then project OA and OB on this line we shall have

Projection of OA $= \text{OA} \cos\theta = \text{OA}\,\frac{n}{\sqrt{n_1^2 + n^2}} = \frac{E_1}{\sqrt{n_1^2 + n^2}}$.

Projection of OB $= \text{OB} \cos(90 - \theta) = \text{OB} \sin\theta = \text{OB}\,\frac{n_1}{\sqrt{n_1^2 + n^2}} = \frac{E_2}{\sqrt{n_1^2 + n^2}}$, since if $\tan\theta = \frac{n_1}{n}$, $\sin\theta = \frac{n_1}{\sqrt{\ {}_1^2 + n^2}}$ and $\cos\theta = \frac{n}{\sqrt{n_1^2 + n^2}}$.

$\therefore$ Projection of OA + Projection of OB

$$= \frac{E_1 + E_2}{\sqrt{n_1^2 + n^2}}$$

Since the sum of the projections is the projection of the sum, or resultant, of OA and OB, which we saw above was an ellipse, we get our final rule.

If two E.M.F.'s exist in two circuits making a right angle with one another and connected in series and having different numbers of turns n and n_1, then the total E.M.F. across the two series circuits is found by adding the E.M.F. per turn in each circuit, at each instant, to form a certain ellipse and projecting this ellipse on a line making an angle $\tan^{-1}\frac{n_1}{n}$ *with that circuit having n turns, and multiplying the result by* $\sqrt{n^2 + n_1^2}$.

CHAPTER V

SERIES TYPE MOTORS

The five different types of motor mentioned above may be obtained by considering the possible permutations of the three elements of which any series-type motor is composed, viz., neutralizing coil, field coil, and armature.

	Neutralizing Coil.	Field Coil.	Armature.
Neutralized series motor	stator	stator	rotor
Repulsion motor	rotor	stator	stator
Inverted repulsion motor . .	stator	rotor	rotor
Induced series motor	rotor	stator	stator
Compensated repulsion motor . .	rotor	rotor	stator

In this classification the "armature" must be regarded as the member into which the power is led by conduction, and the "neutralizing coil" as that in which it is, or may be, induced, while the "field" is that which produces the useful flux. In the case of the inverted repulsion motor, the "field" coil is only a component of the "armature."

If we recollect that machines in which armature and neutralizing coil are on the same member are impossible, it will easily be seen that we have enumerated all possible combinations of these elements, and in fact the induced series motor appears here as a secondary modification of the repulsion motor.

Before discussing individual machines in detail it will be convenient to adopt a few simple conventions as to notation.

In order to make our results quite clear, let us take as a numerical example a small machine of which the following are the particulars:

Number of poles	4
Effective air-gap	·1 cm.
Gap diameter	30 cm.
Core length	15 cm.
Number of primary conductors . . .	304
Turns, per pole	38
Number of secondary conductors . . .	304

To ascertain the relations between the currents, ampere turns, fluxes, and E.M.F.'s, we make use of the well-known fundamental formulæ:

ampere turns $=$ ·8 $\times$ effective single gap length in centimetres $\times$ maximum gap density.

Assuming a current of 5 amperes (R.M.S.)

$$\sqrt{2} \times 5 \times 38 = \cdot 8 \times \cdot 1 \times \text{maximum gap density.}$$

Maximum gap density $= 3360$ per square centimetre.

Assuming, for the present, an harmonic flux distribution, we get

$$\frac{\text{Average gap density}}{\text{Maximum gap density}} = \frac{2}{\pi} = \cdot 636,$$

so that

$$\text{Maximum flux per pole} = \cdot 636 \times 3360 \times \frac{\pi \times 30 \times 15}{4}$$
$$= 755{,}000.$$

Also, the root mean square E.M.F. due to this flux will be at 60 cycles, by the ordinary formula

$$E = 4{\cdot}44 \times \cdot 9 \times 60 \times 152 \times 755{,}000 \times 10^{-8} = 275 \text{ volts.}$$

Since we neglect saturation, all these quantities, viz., the current, ampere turns, maximum flux density, total flux, and E.M.F., are directly proportional to one another. Hence to suitable scales, the same line may represent all of them graphically. In general, it will be most convenient to draw only one diagram representing primarily, say, the E.M.F., and derive the values of the other quantities from this by a change of scale.

For instance, we know that 5 amperes correspond to 275 volts and 755,000 lines per pole. Hence, if we choose a scale of amperes such that, say, 5 amperes correspond to 1 cm. on the diagram, we know at once that 1 cm. must represent 275 volts and 755,000 lines per pole. This procedure we shall adopt.

A further convention, which is important to remember, is that

a vector represents a current and the applied E.M.F. to which it is due, not the back E.M.F. produced by it.

The Repulsion Motor.

The rotor and stator windings of this machine are entirely independent of one another, so that we can assume, without limiting the generality of our investigation, that they have the same number of turns on each. We shall deduce the flux distribution in this machine direct from the principles discussed above.

This flux induces in the stator winding an elliptical distribution of E.M.F., whose projection on the stator axis gives the terminal E.M.F. In the rotor the same flux induces, in

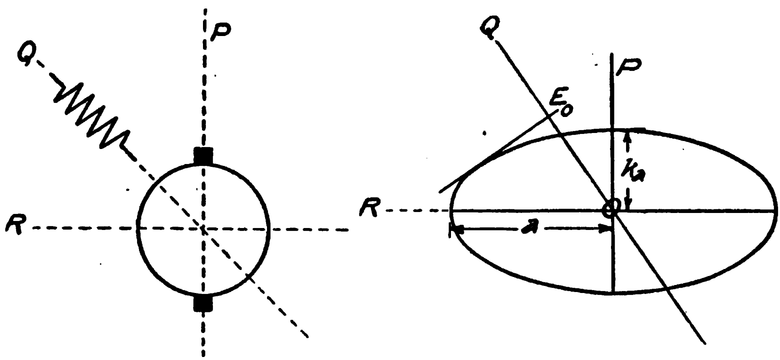


FIG. 54.

accordance with the principles laid down above, the same E.M.F. plus k times its complement, if the machine is running at a speed of k times synchronism.

The deduction of the air-gap flux proceeds as follows:

Since the rotor is short-circuited along the axis OP there can be no E.M.F. along that axis, and the rotor E.M.F. ellipse is therefore a straight line along OR perpendicular to OP. In order to give rise to such a rotor E.M.F. distribution at speed k, we require a flux ellipse whose axes lie along OR and OP; that along OR being of length a, say, and that along OP being ka, in which case, as may be easily seen, the "transformer" E.M.F. and E.M.F. of rotation along OP will cancel (p. 45), leaving a straight line distribution of E.M.F. The E.M.F. due to this flux distribution

in the stator is, in accordance with the conventions adopted above represented by the same ellipse as the flux (Fig. 54) (of course, with the phase-arrow displaced a quarter period).

The projection of this ellipse on the stator axis OQ, therefore, must be constant and equal to the terminal E.M.F.

Hence we get the following rule for determining the flux distribution of the ideal repulsion motor:

(1) Draw a line perpendicular to OQ and at a distance from the origin equal to the maximum terminal E.M.F.

(2) Tangent to this line draw an ellipse whose axes are OP and OR and the ratio of the axis OP to the axis OR is k. This will be the flux ellipse at any speed.

The Neutralized Single-Phase Series Motor.

This machine consists of a field and neutralizing winding placed upon the stator and connected in series with the armature, so that the neutralizing winding exactly annuls the magnetomotive force of the armature, there being no resultant flux along the brush axis OR (Fig. 2).

We shall suppose that there are a times as many turns between the brushes as there are in the field.

As a consequence of the presence of the neutralizing coil, the flux ellipse is a straight line along OP, which induces in the field an E.M.F. e equal to it in accordance with the conventions we have adopted, and in the armature an E.M.F. $e_1 = ake$, in quadrature with e.

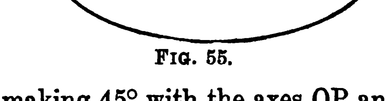


FIG. 55.

Compounding e and e_1, we obtain the E.M.F. ellipse, which corresponds to the ellipse of E.M.F. distribution round the commutator when $a = 1$. A tangent to this ellipse (p. 46), making 45° with the axes OP and OR, gives the terminal E.M.F. (see Fig. 55).

Hence the rule for determining the E.M.F. ellipse in the single-phase series motor is:

(1) Draw a line making an angle of 45° with OP and OR and at a distance from the origin equal to the root mean square terminal E.M.F.

(2) Tangent to this line, draw an ellipse with axes OP and OR and having the axis OR equal to ak times the axis OP.

This ellipse will be the ellipse of resultant E.M.F. in the machine. The position of the phase-arrow on it must agree with the assumed phase-arrow on the terminal E.M.F.

The current in the motor will be proportional to and in quadrature with the projection of this ellipse on OP.

The Inverted Repulsion Motor.

This machine is constructed exactly like the ordinary repulsion motor save that the stator winding is short-circuited and the rotor winding connected to the line.

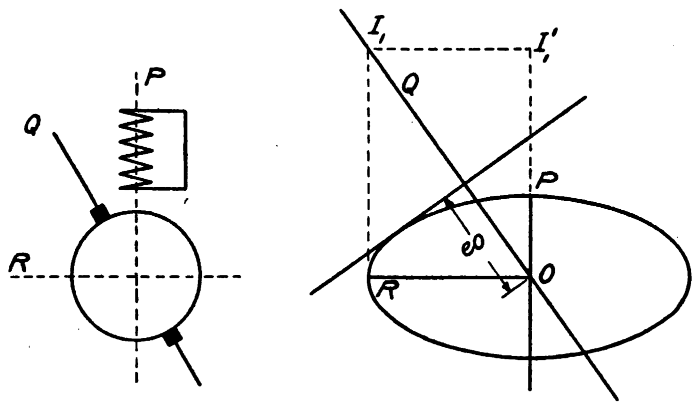


FIG. 56.

Since the stator winding is short-circuited the flux along OP, now the stator axis, is always zero and the flux ellipse reduces to a straight line along OR, just as the rotor E.M.F. ellipse did in the ordinary repulsion motor (see Fig. 56).

This purely pulsating flux ellipse induces in the motor an E.M.F. e equal and in quadrature with itself along OR and $e_1 = ke$ along OP in quadrature with e (p. 42). Compounding e and e_1, we get the ellipse of rotor E.M.F., whose axis OP is, as usual k times its axis OR.

This ellipse must touch a line perpendicular to OQ and at a distance e_0 from the origin equal to the maximum terminal E.M.F.

So we see that the distribution of rotor E.M.F. in the inverted

repulsion motor is identical with that of stator E.M.F. in the ordinary repulsion motor. The primary, or rotor current, is such that its projection on OR is equal to, but in quadrature with, the projection of the E.M.F. ellipse on that axis. The secondary or stator current is equal and opposite to the projection of the primary current on OP. This construction is also true for the ordinary repulsion motor.

The Compensated Repulsion Motor.

This machine carries a single distributed, single-phase winding on the stator which is connected in series with a pair of rotor brushes displaced 90 electrical degrees from the axis of the stator windings. A pair of short-circuited brushes is arranged co-axially with the stator winding (see Fig. 13).

The machine only differs from a modification of the ordinary repulsion motor, in which the single stator coil, which is usual, is resolved into two coils at right angles, in that the coil at right angles to the short-circuited brushes is arranged upon the rotor instead of the stator.

Thus the flux distribution corresponding to a given current is identical with that of the ordinary repulsion motor, but the terminal E.M.F. corresponding to that current is different. We did not find it necessary in the ordinary repulsion motor to calculate the value of the secondary E.M.F. or E.M.F. along OR. Since in the compensated repulsion motor, however, this E.M.F. occurs in the primary circuit, we must now proceed to calculate it.

In Fig. 57 is shown the ellipse of flux distribution, which is also the ellipse of stator E.M.F., exactly as in the repulsion motor, and we saw that the "transformer" and "rotation" E.M.F.'s along the axis OP cancelled one another.

Along the axis OR (Fig. 57), however, the "transformer" E.M.F. will be a, equal to the semi-axis of the ellipse.

The "rotation" E.M.F., however, will be k times ka the axis of the ellipse along OP. It will be exactly opposed to the transformer E.M.F., hence the resultant E.M.F. along OR is $(1 - k^2)a = \text{OR}$.

If we put $\dfrac{\text{turns in series on stator winding}}{\text{turns in series between brushes}} = b$, the stator E.M.F. will be b times OP′, or kba.

In order to get the terminal E.M.F., we have to compound the stator E.M.F. along OP with the rotor E.M.F. along OR, which is in quadrature with it, in exactly the same way as we did with the neutralized series motor.

As in the series motor, too, the ellipse obtained by compounding these two quantities must touch a line making an angle of 45° with OP and OR (see p. 46) and at a distance from the origin equal to the root mean square terminal E.M.F. The axes of this ellipse of resultant E.M.F. are kb and $(1 - k^2)\,b$. Hence the rule for constructing the diagram of this motor is:

(1) Draw a line making an angle of 45° with the axes OP and OR and at a distance from the original equal to the R.M.S. terminal E.M.F.

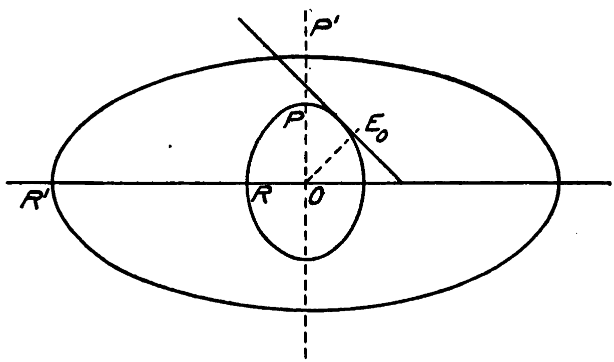


FIG. 57.

(2) Tangent to this line, and with axes OP and OR, draw an ellipse the length of whose axes are in the proportion kb and $(1 - k^2)b$ along OP and OR respectively. This will be the ellipse of resultant E.M.F.

(3) Having drawn this ellipse with axes OP and OR, draw another with axes OP′ = OP/b and OR′ = OP/kb. This will be the flux distribution ellipse.

Having obtained this, we may calculate the current from it in exactly the same manner as in the repulsion motor.

The Induced Series Motor.

This machine consists of an inducing primary winding on the stator and a field winding at right angles to it, also on the stator.

On the rotor we have two brushes arranged co-axially with the stator winding and connected in series with the field winding. Since rotor and stator are quite independent, we shall assume that the primary and field windings have the same number of turns and the number of armature turns is a times the field turns.

The E.M.F. induced by the primary flux in the secondary is equal to the resultant of the E.M.F. induced in the field coil and the rotation E.M.F. in the armature.

We must therefore construct first this ellipse of resultant E.M.F. just as we did in the neutralized series motor. Its axes will be along OP and OR, and if the length of that along OR be e, that along OP will be ake.

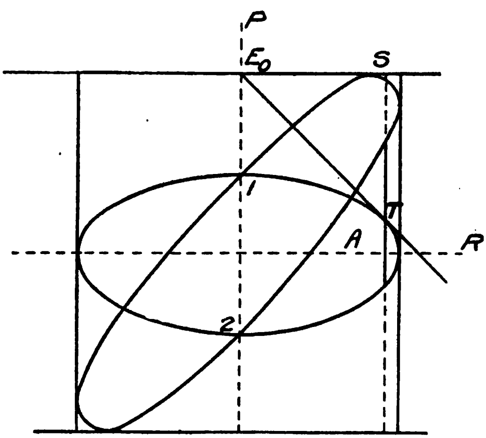


FIG. 58.

Let e_0 be the maximum impressed E.M.F. Draw a line perpendicular to OP (Fig. 58), and at a distance e_0 from the origin. Through the point E_0, where this line cuts the axis OP, draw another E_0T, making 45° with the axes OP and OR. The shortest distance between this line and the origin is easily seen to be $e_0/\sqrt{2}$.

Draw the ellipse of resultant E.M.F. with the ratio

$$\frac{\text{semi-axis OP}}{\text{semi-axis OR}} = ak$$

and tangent to the line just drawn at 45° to both axes.

To get the flux distribution, we have to compound the component along OR of the ellipse just drawn, which represents the "transformer" E.M.F. along OR, with the E.M.F. OE_0. When e_0 is a maximum, the phase-arrow on the vector ellipse, just drawn, is at the point of tangency of the ellipse with the line E_0T.

This gives us one of the data we require for drawing the ellipse

of flux distribution. The projection of this ellipse on the axis OP is the impressed E.M.F. e_0, and its projection on the axis OR is the same as the projection of the ellipse of resultant secondary E.M.F. already drawn.

Hence, it is tangent to the line E_0S already drawn perpendicular to OP and at distance e_0 from the origin.

Draw a line perpendicular to OR and tangent to the secondary E.M.F. ellipse. The ellipse of flux distribution will be tangent to this line also. We thus have two pairs of parallel tangents to the ellipse.

Project the point T on to the line E_0S by a line perpendicular to OR and cutting E_0S in S. This is the point where the flux distribution ellipse touches E_0S. For, when E_0 is a maximum, the phase-arrow of the secondary E.M.F. ellipse is at T and its projection on OR is the point A where ST cuts OR.

Compounding OE_0 and OA, we get OS, which gives us the radius vector of the flux ellipse when OE_0 is a maximum.

To sum up, in order to draw the flux distribution ellipse of the induced series motor we must perform the following constructions:

(1) Draw two lines E_0S at distances E_0 from the origin on either side and perpendicular to OP.

(2) From the point E_0 draw E_0T, making 45° with both axes.

(3) Draw an ellipse with ratio $\dfrac{\text{semi-axis along OP}}{\text{semi-axis along OR}} = ak$ and touching E_0T at T.

(4) Draw two lines touching this ellipse and perpendicular to OR.

(5) Draw through T and perpendicular to OR a line cutting E_0S in S.

(6) Draw an ellipse tangent to the four lines we have drawn perpendicular to OP and OR and tangent to E_0S in S.

This is the ellipse required.

By another line of reasoning we can show that the axis along OP of the secondary E.M.F. ellipse is also a diameter of the flux distribution ellipse. Because, when the secondary E.M.F. along OR is zero, the terminal E.M.F. must be equal to the radius vector of the secondary E.M.F. ellipse along OP.

CHAPTER VI

SINGLE-PHASE, SHUNT TYPE MOTORS

There is by no means so large a variety of shunt type as of series type motors having different modes of operation though substantially the same characteristics. Although the number of constructional modifications which may be suggested is, of course, endless, all practicable types of motor reduce, in the end, to what is essentially one machine. This is due to the following facts.

Consider the ordinary shunt motor operated on alternating current. If the armature current is approximately in phase with the line E.M.F., which it should be, of course, for operation on good power factor, it will be in quadrature with the flux which lags 90° behind the line E.M.F. In order, therefore, to keep flux and current in phase we need to supply the field with an E.M.F. leading 90° on the armature or line E.M.F.

There is but one way to generate such an E.M.F. if we are to retain a purely single-phase motor, and that is by means of a pair of brushes arranged on the armature at right angles to the load brushes or those which carry the main current. The practical machine in which this principle is carried out in its simplest form is usually called the Atkinson commutator induction motor (see Fig. 16). It consists of a distributed single-phase winding on the stator, a pair of short-circuited brushes co-axial with the stator winding, and another pair at right angles thereto, also short-circuited.

The E.M.F. of rotation induced in this latter pair of brushes by the primary flux leads 90° on the line E.M.F. and is therefore capable of producing the field we require. This commutator armature with its two pairs of short-circuited brushes may be replaced by a plain squirrel cage armature giving us the ordinary single-phase induction motor.

Let us consider, in the light of the principles discussed above, what must be the flux distribution of such a machine.

Since the rotor is short-circuited along all axes, the rotor E.M.F. must be zero along all these axes.

Now there is only one flux distribution which can induce zero voltage in a revolving armature, and at the same time a voltage e_0 along one axis of the stator, and that is a purely circular or rotating flux distribution. It can only do this, moreover, at one particular speed, viz., synchronism. Hence, the ideal induction motor, devoid of losses and leakages, can only run at one speed, viz., synchronism.

Hence, to draw the flux distribution of the single-phase induction motor, we get the following rule:

(1) Draw a line perpendicular to the stator axis OP at a distance from the origin equal to the maximum E.M.F. e_0.

(2) Draw a circle tangent to this line and this will be the flux distribution at synchronous speed.

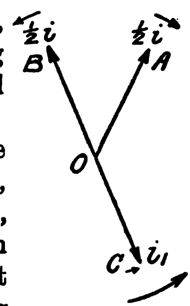


FIG. 59.

This circle represents, to the appropriate scale, not only the flux distribution, but the magnetizing current, which is the resultant of the rotor and stator currents.

Suppose the machine runs clockwise. The stator current is purely pulsatory or single-phase, on account of the nature of the stator winding, and hence requires supplementing by a certain rotor current in order to give a resultant rotating current. The stator current being pulsatory and of magnitude i, say, may be resolved into two equal, oppositely rotating currents each of magnitude $\frac{1}{2}i$.

If the rotor current is $i_1 = \frac{1}{2}i$, and is opposite in phase to that component OB of the stator current which revolves counter-clockwise, or against the revolution of the motor, it will cancel it at every instant, leaving merely the component OA to supply the flux. The magnitude of OA must therefore be sufficient to produce the flux, and therefore the total stator current i must have twice this magnitude. Thus we get the following results relating to the magnetizing currents of the single-phase induction motor:

(1) The rotor current is a purely revolving current, revolving

against the direction of rotation of the motor, and is equal in magnitude to the magnetizing current necessary to produce the flux, calculated in the ordinary way.

(2) The stator current is twice as great as the rotor current, and is, of course, a purely pulsatory one.

If the stator current were zero and the rotor current revolved synchronously with the flux, it would supply the necessary magnetomotive force just as well as the above current distribution, with the additional advantage of taking no current from the line.

A slight change in the Atkinson commutator induction motor known as "phase compensation" is often employed to bring about this result.

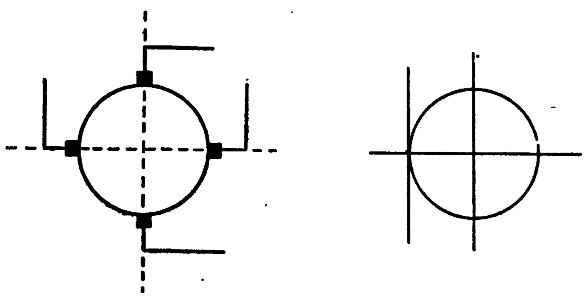


FIG. 60.

This is shown in Fig. 22. A small E.M.F. in phase with the line E.M.F. is introduced into the brushes perpendicular to the stator axis. We have now to consider how this produces the result mentioned above.

If we can by any means produce a synchronously revolving current of the proper magnitude in the rotor, the stator current will automatically disappear.

Consider for a moment a commutator rotor (Fig. 60) into which is conducted a balanced polyphase current of any frequency, the rotor revolving synchronously with the current. Whatever the magnitude of the flux may be, since it cannot cut the rotor, it can induce no E.M.F. therein. The current, therefore, due to a polyphase E.M.F. e is simply $i = \frac{e}{r}$ determined by the resistance according to Ohm's law.

Hence we conclude that we can produce a synchronously revolving current of any desired magnitude in the rotor of our machine by feeding in a small polyphase E.M.F. through the brushes. We saw that the current in the XX axis, or brush circuit perpendicular to the stator axis, was in phase with the line E.M.F., so the E.M.F. to be fed into that circuit should be a part of the line E.M.F.

We have now to investigate how it is that we can dispense with the E.M.F. fed into the YY axis parallel to the stator axis.

Here we have an E.M.F. e_1 in phase with e_0 fed into the XX axis. Our problem is to ascertain the effect of this on the flux distribution.

Since the rotor E.M.F. has to balance the E.M.F. e_1 conducted in at the XX brushes, it is clear that the flux distribution can no longer be circular, since a circular or purely rotating flux, as we saw, produces no E.M.F. in the rotor.

Any elliptical flux distribution whatever induces a purely rotating E.M.F. in a rotor running at synchronism, because any such distribution can be resolved into two oppositely rotating fluxes, of which that revolving with the rotor can induce no E.M.F. Hence, whatever the flux distribution, the rotor E.M.F. will be circular and revolve against the direction of rotation of the machine. To the scale which we are using, the rotor E.M.F. will be represented by a vector *twice* as long as the flux vector, since the rotor runs at twice synchronism relative to the flux.

Hence, to get the rotor E.M.F. for the motor under consideration, we get the following rule:

> Draw a line perpendicular to OR and at a distance e_1 from the origin. Tangent to it draw a circle the position of the phase-arrow on which is its point of tangency with the line just drawn (see Fig. 60).

Along the axis OR this rotor E.M.F., induced by the flux, is balanced by the impressed E.M.F. conducted in through the brushes. Hence the current in this axis remains as before, merely a magnetizing current sufficient to produce the flux along the axis (now slightly changed, however).

Along the axis OP, however, this induced rotor E.M.F., which we may note is in this axis in quadrature with the line E.M.F., is entirely unbalanced by any other, and hence, in an ideal machine such as we are considering, would give rise to an infinite

current. Hence we must conclude that it is not permissible to conduct a finite E.M.F. into an ideal rotor of this type at synchronism.

Here we must introduce the resistance of the circuit along the YY axis in order to limit the current along that axis. The same thing has to be done, of course, when considering polyphase compensation, as we did above. Let r be the resistance between the YY brushes.

We then get in the YY axis a current e_1/r in quadrature with the primary E.M.F., that is, in phase with the primary magnetizing current. This current, or "YY compensating current," may be given any magnitude and caused either to lag or lead on e_0, according to the magnitude and sign of e_1.

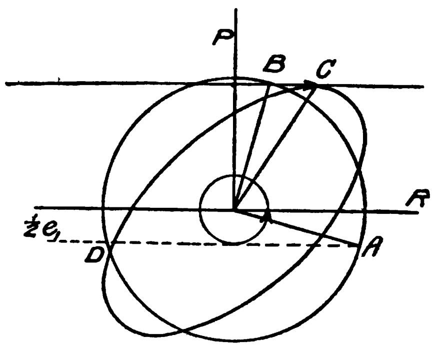


FIG. 61.

It may therefore, by a proper adjustment of e_1, be caused to supply the whole of the magnetizing current necessary in the YY axis and thereby reduce the stator current to zero.

When we have done this we have reversed the current which flows in the YY axis of the ordinary induction motor, that is, the rotating current in the armature is now going *with* the rotor, not against it.

It will be of interest now, having ascertained to what the compensating effect is due, to calculate the flux distribution at synchronous speed in the compensated motor. We can do this most easily by considering the stator E.M.F. ellipse as composed of two oppositely rotating E.M.F.'s, of which that one which rotates counter-clockwise, or against the direction of rotation of the motor, is alone responsible for the rotor E.M.F. The E.M.F. produced by the counter-clockwise flux is twice as great in the rotor as in the stator, so that as we are considering stator E.M.F.'s the magnitude of the flux will be $\frac{1}{2}e_1$ if e_1 be the compensating voltage. Thus we have already found the oppositely rotating component of the ellipse and have, therefore, only to find the

clockwise rotating component. If OP (Fig. 61) be the primary axis, then the stator E.M.F. ellipse must touch a line drawn perpendicular to this axis at a distance e_0 from the origin.

When e_0 is a maximum, e_1 is a maximum also, and hence, the counter-clockwise rotating vector lies along OR at this instant since it is into OR that e_1 is fed.

Let OC be the radius vector of the ellipse at the point where it touches the line parallel to OR. $OC = OB + BC$, vectorially, if OB is the clockwise rotating E.M.F. and BC the counter-clockwise. Since we know that BC is parallel to OR, it follows that the projection of OB on OP is also e_0, or that its extremity lies on the same line to which the ellipse is tangent. A quarter

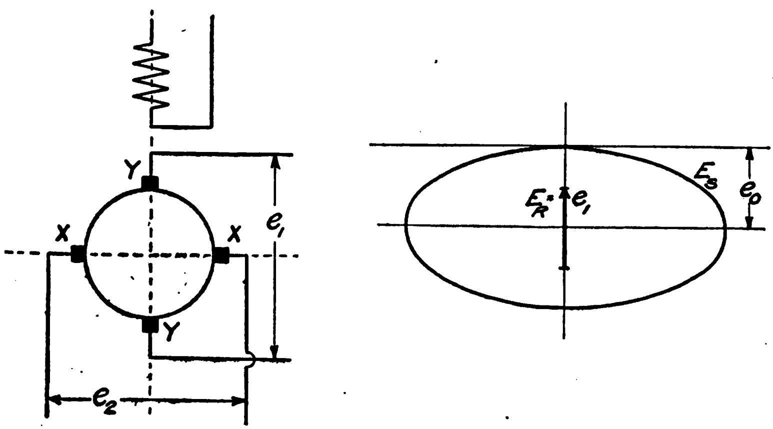


FIG. 62.

period later the counter-clockwise E.M.F. lies along OP, and since the primary E.M.F. is zero, the clockwise-rotating E.M.F. must have a component along OP equal and opposite to the counter-clockwise.

Hence we get our final construction for the clockwise E.M.F. Draw a line AD parallel to OR and on the opposite side of it from the line BC, its distance from OR being $\frac{1}{2}e_1$. Describe a circle from the origin as centre cutting BC in B and AD in A. Let the diameter of the circle be such that OB is perpendicular to OA and it will satisfy the conditions enumerated above, and therefore be the clockwise component

of the desired stator E.M.F. ellipse. Combining this with the counter-clockwise component which we already know, we can draw the ellipse by the usual constructions.

We now come to the generalized type of shunt machine (Fig. 62), in which E.M.Fs are fed into the rotor brushes for the purpose of varying the speed. In order to vary the speed, the E.M.F. fed into the brushes parallel to the stator axis should be in phase with the line voltage and that fed into those perpendicular thereto should be in quadrature.

Consider, first, the case where the XX brushes are short-circuited and an E.M.F. e_1 is fed into the YY brushes and e_0 to the primary. The conditions to be satisfied by the flux ellipse are as follows: it must produce zero rotor E.M.F. along the XX axis, and an E.M.F. e_1 along the YY, and at the same time an E.M.F. e_0 in the stator.

We saw in discussing Fig. 52 that if a is the YY and b the XX axis of a flux ellipse we must have $k = \frac{b}{a}$ or $ka = b$ in order that the XX axis of the resulting E.M.F. ellipse may be reduced to zero.

We know the flux along the YY axis, or a, since it must be such as to balance the primary E.M.F. Hence according to our convention $a = e_0$. This assumes the same number of turns on rotor and stator of course. Since $b = ka = ke_0$ we now know the complete flux distribution. The flux along XX induces a counter E.M.F. along the YY axis equal to k^2a or k^2e_0 (k times its own magnitude ka). Hence the resultant rotor E.M.F. is

$$(1-k^2)e_0.$$

This must be equal to e_1 so we get

$$e_1 = (1 - k^2)\, e_0$$

which, since e_1 and e_0 are arbitrary, can only be true for one particular value of k which is the no-load speed of the motor.

The machine is, therefore, a constant speed motor which can only run in the immediate neighbourhood of this one particular speed.

To sum up, in order to draw the flux ellipse of this type of motor, we proceed as follows:

Calculate k from the equation $e_1 = (1 - k^2)\, e_0$. Draw an ellipse whose axes lie along the XX and YY axes and whose axis along YY is equal to e_0, and its axis along XX is ke_0 (see Fig. 62).

Similarly, we can draw the flux ellipse if an E.M.F. is conducted into the XX axis (Fig. 19) the YY axis being short circuited. Here the conditions which determine our ellipse are:

(1) The axis of the ellipse along YY (or OP) must be k times the other axis.

(2) It must be such as to balance e_0.

(3) k is still determined by the equation $(1 - k^2)\, e_0 = e_1$.

So that the construction for determining the ellipse will still be the same as before, save that the minor axis will now be where the major axis was before (see Fig. 63).

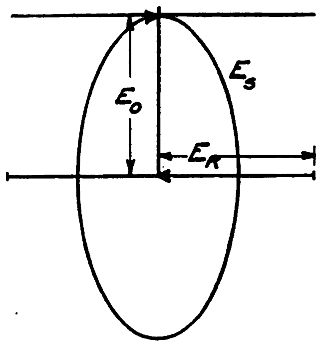


FIG. 63.

A more general manner of determining k and the flux distribution is the following:—

(a) We saw (p. 43) that if the semiaxes of the standstill E.M.F. are a, b, those of the E.M.F. during running will be $a \pm k\, b$ and $b \pm k\, a$.

(b) We know that $a = e_0$.

From the three equations

(1) $e_0 = a$

(2) $e_1 = a \pm k\, b$

(3) $e_2 = b \pm k\, a$

we can determine a, b, and k.

CHAPTER VII

A MORE EXACT THEORY OF THE SERIES TYPE MOTORS

DEFINITIONS AND PRELIMINARY INVESTIGATIONS

In the following chapters we shall endeavour to discuss the five fundamental forms of series type motor in sufficient detail to take into account the magnetic leakages, etc.

Before doing so, however, it will be necessary to explain the principles on which these discussions will be based somewhat more fully.

In order to carry out our investigation satisfactorily we require first a clear and systematic notation for the various vectors of our diagram.

We have to consider three classes of quantities, E.M.F.'s, fluxes, and currents or ampere turns.

Our notation should show clearly the following facts :

(1) Whether the vector denoted represents E.M.F., flux, or ampere turns.

(2) At which of the two instants considered it exists.

(3) Whether it represents an effect in the rotor, stator, primary secondary, or tertiary (commutating coil).

(4) When there are several vectors acting in the same circuit, we shall require distinguishing marks for each.

To fulfil these conditions we adopt the following notation:

$t = 0$	$t = 1$
E	F are E.M.F.'s at the two instants $t = 0$ and $t = 1$, a quarter period apart.
I	J are M.M.F.'s a quarter period apart.
M	N are fluxes a quarter period apart.

Thus to get the ellipse of E.M.F. we take the resultant of all E.M.F.'s, E and all E.M.F.'s F as the two conjugate axes of our

ellipse. The suffix $_{S}$, $_{R}$, as M_S, M_R, indicates that the flux interlinks the stator or rotor respectively. The suffix $_1$, $_2$, $_3$, as in I_1, I_2, I_3, indicates that the M.M.F. I is due to the current in the primary, secondary, or commutating coil circuit, respectively.

The further suffix $_a$, $_b$, $_c$, as in E_{2a}, indicates that there are several E.M.F.'s (at $t = 0$) in the secondary circuit which require distinction. We shall not have much occasion for these double suffixes.

Every elliptical field machine may be investigated by studying the phenomena which occur along two independent axes, making an angle with one another in space. Along each of these axes we shall have in general a component of the stator circuit, and also of the rotor circuit. The two axes are made interdependent by E.M.F.'s of rotation appearing in one axis due to the fluxes

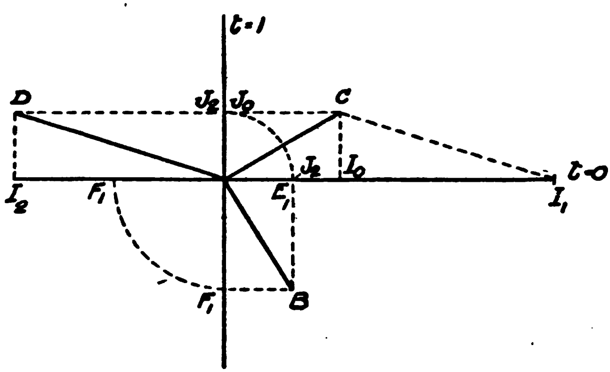


FIG. 64.

along the other and also, perhaps, because we may have the same stator or rotor current in each. Each of these axes we may treat as a transformer, all E.M.F.'s of rotation, as well as the external E.M.F.'s being treated as impressed E.M.F.'s.

It will be useful to develop this point of view more fully.

Let us consider first the phase diagram of an ideal transformer without losses and leakages, and see how we may construct a linear diagram from it to describe the phenomena in one axis of our machine.

In such a transformer let OI_1 be the primary current and OB the primary E.M.F., both being supposed given.

We shall use the same conventions as to the current, E.M.F., and flux scale, as those explained in Chapter V.

According to this convention if OB represents the E.M.F. the magnetizing current will be represented by a vector OC of *equal* magnitude to OB but lagging 90° behind it. The secondary current will therefore be CI_1 or OD. Now let us consider how we may represent the facts given by this phase diagram by means of a purely linear diagram.

To do this we make use of the conception mentioned above (Chapter III.) in which the linear diagram is regarded as an edge view, so to speak, of the phase diagram.

We have first to determine the values of the various quantities OI_1, OB, OC, and OD at two instants a quarter period apart. To ascertain this we must project our vectors on two mutually perpendicular axes, which may conveniently be taken parallel and perpendicular to OI_1.

According to the system of notation described above the components of the above vectors will be denoted as follows:

Phase diagram vector.	Magnitude when t=0.	Magnitude when t=1.
OI_1	I_1	$J_1\ (=0)$
OB	E_1	F_1
OC	I_0	J_0
OD	I_2	J_2

All these components must now be swung into a given axis by a rotation in a definite direction (say clockwise), as has been done in the diagram (Fig. 64), where the line corresponding to $t = 0$ is the linear diagram we require.

It is geometrically evident that since OC is equal and perpendicular to OB_1 we have $OI_0 = -OF_1 = OI_1 + OI_2$.

It is also clear that

$$J_2 = E_1, \text{ or more generally,}$$

$$J_1 + J_2 = E_1 \text{ with } J_1 = 0.$$

The two equations

$$J_1 + J_2 = E_1 \text{ and } I_1 + I_2 = -F_1$$

are quite sufficient in themselves as a foundation for our linear diagram without any reference to the phase diagram.

To prove this we proceed as follows:

Set out on the linear diagram I_1, E_1 and F_1, which are supposed given together with $J_1 = O$.

Then the above equations enable us to determine I_2 and J_2 directly as follows:

$$I_2 = I_1 - F_1,\ J_2 = E_1.$$

At first sight it looks curious to see currents such as I_1 and E.M.F.'s such as F co-existing in the same equation, but it must be remembered that these equations depend strictly on our convention identifying the magnitudes of our E.M.F.'s and the corresponding magnetizing currents.

FIG. 65.

In more complex problems, such as we shall deal with later, it will be convenient to tabulate these calculations. This may easily be done as follows:

	Flux.	Current.	E.M.F.	Flux.	Current.	E.M.F.
Primary .	$M_1 = -F_1$	I_1	E_1	$N_1 = -E_1$	$J_1 = O$	F_1
Secondary	$M_2 = -F_1$	$I_2 = F_1 - I_1$	$-E_1$	$N_2 = N_1$	$J_2 = E_1$	$-F_1$

To illustrate the application of these phase diagrams and corresponding linear diagrams we will suppose that OI_1 and OB are the current and E.M.F. in the primary of an ideal repulsion motor, and endeavour to find the secondary current, flux distribution and speed.

In the diagram (Fig. 66), let OP, the primary axis, be taken as the axis of the transformer which we have just studied.

The linear diagram which we have just developed will apply to this axis without modification. It is reproduced in Fig. 66.

The secondary current along the axis OP, however, is now only a component of the true secondary current along its own axis. We shall accordingly denote the components of the secondary current along OP by J''_2 and I''_2.

Projecting these across on to the rotor axis OQ by lines perpendicular to OP we get I_2 and J_2 the true secondary currents when $t = 0$ and $t = 1$.

Compounding I_1 with I_2 vectorially we get I_0 the resultant magnetizing current when $t = 0$. Since $J_1 = 0$ we have $J_2 = J_0$, the resultant magnetizing current when $t = 1$. The vectors I_0 and J_0 now form the conjugate axes of our flux or M.M.F. ellipse, which we can immediately proceed to draw. The ratio of the semi-axis of this ellipse parallel to OQ to that perpendicular thereto will now be k, the ratio of the speed to synchronism.

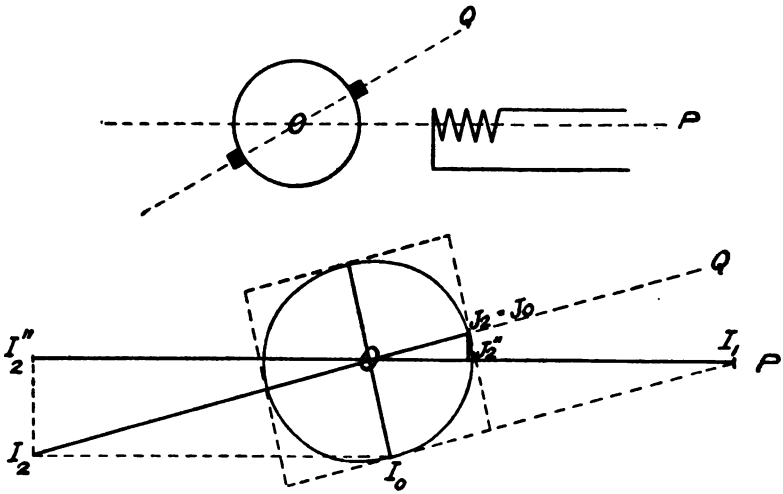


FIG. 66.

From this point of view it is quite possible to develop a complete theory of single-phase motors, taking all losses and leakages into account. It has the advantage of showing clearly the relation between the space and the phase diagram. In the following chapter, however, we shall adopt a somewhat simpler view though still retaining the notation of the present chapter and the method of considering the occurrences at two instants $t = 0$ and $t = 1$ a quarter period apart.

CHAPTER VIII

THE EFFECTS OF MAGNETIC LEAKAGE ON THE SERIES TYPE MOTORS

We shall now endeavour to introduce into our argument the leakage effects which occur in each motor. In order to do this without complicating the argument more than absolutely necessary we shall first discuss leakage effects in general.

In every machine there are three distinct fluxes.

(1) The main flux crossing the gap and interlinking both rotor and stator conductors, proportional to the resultant of the stator and rotor currents.

(2) The stator leakage flux interlinking the stator only, and proportional to the stator current only.

(3) The rotor leakage flux interlinking the rotor, and proportional to the rotor current only.

These three fluxes are physically distinct, but (1) and (2) both interlink the stator circuit and (1) and (3) both interlink the rotor circuit, producing E.M.F.'s therein that can be added vectorially. Therefore, we may represent these leakage E.M.F.'s by vectors or ellipses in the same way as any other E.M.F.'s. Since the leakage fluxes at constant frequency bear a definite ratio, as given in Chapter V., to these E.M.F.'s we may represent these also by vectors.

The leakage flux corresponding to any current will in general be proportional to that current and in phase with it in time. It will, moreover, bear a constant ratio to the air-gap flux produced by the same current. In Chapter V. we adopted a convention whereby the same vector ellipse represented a current or the flux or M.M.F. corresponding to it, to different scales. We shall continue this convention here and shall assume that the leakage flux and E.M.F. bears a constant ratio C to the air-gap flux or E.M.F. due to the same current.

Hence, if a current be represented by a vector I its leakage E.M.F. will be of magnitude CI. There will in general be two of

these leakage coefficients or factors, the stator leakage coefficient C_1 and the rotor leakage coefficient C_2.

Hence any ellipse of current or current vector, whether in stator or rotor, is accompanied by a leakage E.M.F. C times as great. In the stator this leakage E.M.F. ellipse will be in quadrature with the current ellipse. If we have a given current ellipse flowing through the brushes of a commutator rotor, then at standstill we shall have a leakage E.M.F. ellipse C_2 times as large. But when the rotor gets into rotation the leakage flux due to the current begins to cut the rotor conductors, and we get leakage E.M.F.'s of rotation. In fact, when running at speed k we shall have, in accordance with the results of Chapter IV.:

The leakage E.M.F. induced in a rotor revolving at speed k *by any leakage flux is equal to the standstill E.M.F. plus or minus* k *times its complement.*

The next points we must note are the following definitions:

(1) The total stator flux is the resultant of the air-gap and stator leakage flux.

(2) The total rotor flux is the resultant of the same air-gap flux and the rotor leakage flux.

Corresponding to each we may draw a total rotor and a total stator flux ellipse.

In the investigation we have just completed relating to ideal motors without losses or leakages, we had only to draw a single ellipse of E.M.F. as no E.M.F.'s existed save those due to the air-gap flux. We shall now find that this single ellipse sub-divides into three, which we may call,

(*a*) The resultant rotor E.M.F. and flux ellipses.

(*b*) The air-gap flux ellipse.

(*c*) The resultant stator E.M.F. and flux ellipses.

With these preliminaries we may proceed to the detailed discussion of the various types.

The method of treatment, therefore, that we shall adopt, will be to assume a certain line current and draw a diagram, showing the leakage E.M.F.'s at standstill. We shall find that all our different types have practically identical diagrams when not running. We shall then consider the machine to be running at speed k, and shall investigate what further E.M.F.'s arise due to the same current.

We shall occasionally find it desirable to make use of the following idea which is very often used in works on alternating currents.

If we have two currents whose M.M.F.'s partially oppose one another, forming as it were the primary and secondary of a transformer, the actual flux of course will be due to the resultant of these currents; but it is quite permissible to consider each current as producing *its own* flux proportional to itself, and taking into account the saturation factor of the resultant flux if necessary. We can then compound these two ideal fluxes to get the resultant flux instead of compounding the currents directly.

We shall find it convenient to divide the five series type motors we wish to discuss into two classes—(1) the repulsion type, including the ordinary, inverted, and compensated repulsion motors, and (2) the series type, including the neutralized and induced series motors. These will be treated as far as possible in parallel columns, so as to bring out their essential resemblances.

The Repulsion Motors.

There are three variations of the repulsion motor, known as the ordinary, inverted, and compensated repulsion motors and shown in Figs. 9, 11, and 13.

The inverted type differs from the ordinary merely in that the stator is short-circuited, while the rotor is connected across the line, while in the compensated type the single primary coil often used in these two types (see Figs. 54 and 56) is resolved into two components, one parallel and the other perpendicular to the short-circuited axis, the latter being placed on the rotor.

At starting, the three are identical and the only differences between them are those due to the different manners in which the E.M.F.'s of rotation are induced.

We shall treat the ordinary and the inverted repulsion motors in parallel columns to bring out the analogies and differences between them and shall then discuss the compensated type.

The Repulsion Motor.

As in Chapter V. the resultant rotor flux will be OM_r (see Fig. 67) perpendicular to OP, since OP is short-circuited.

The Inverted Repulsion Motor.

As in Chapter V., the resultant stator flux will be OM_s (see Fig. 68) perpendicular to OP, since OP is short-circuited.

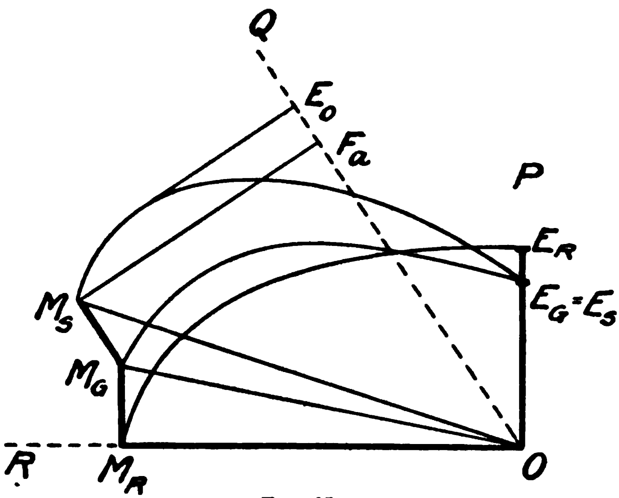


FIG. 67.

The air-gap flux will have a component M_gM_r along OP necessary to induce an E.M.F. balancing the rotor leakage E.M.F.

Hence OM_g will be the air-gap flux. The resultant stator flux will have a further component M_gM_s along OQ balancing the stator leakage. All these E.M.F.'s will be in phase in time, and therefore will be zero at the alternate instant. Projecting OM_s on OQ the terminal E.M.F. will be OF_a, equal and in quadrature with this projection.

Running at speed k with current unchanged.—All the E.M.F.'s and fluxes discussed above remain unchanged. As we saw, OM_r, OM_g, and OM_s are the rotor, air-gap, and stator

The air-gap flux will have a component M_gM_s along OP necessary to balance the stator leakage E.M.F.

Hence OM_g will be the air-gap flux. The resultant rotor flux will have a further component M_gM_r along OQ due to the rotor leakage. All these E.M.F.'s will be in phase in time, and therefore will be zero at the alternate instant. Projecting OM_r on OQ the terminal E.M.F. will be OF_a, equal and in quadrature with this projection.

Running at speed k with current unchanged.—All the E.M.F.'s and fluxes discussed above remain unchanged. As we saw, OM_g was the air-gap flux producing an equal E.M.F. OF_g at standstill. It will now

fluxes respectively when $t = 0$, but all these fluxes are no longer zero when $t = 1$. On the contrary, OM_g will now induce an E.M.F., $OE^1_2 = kOM_g$ in the rotor perpendicular and in quadrature with itself, which has the component induce in addition an E.M.F. in the rotor $OE_r = kOF_g$ at right angles to it in space, and in quadrature in time. OE_r and OF_r will therefore form two conjugate diameters of our ellipse of resultant rotor E.M.F. which we can at once proceed

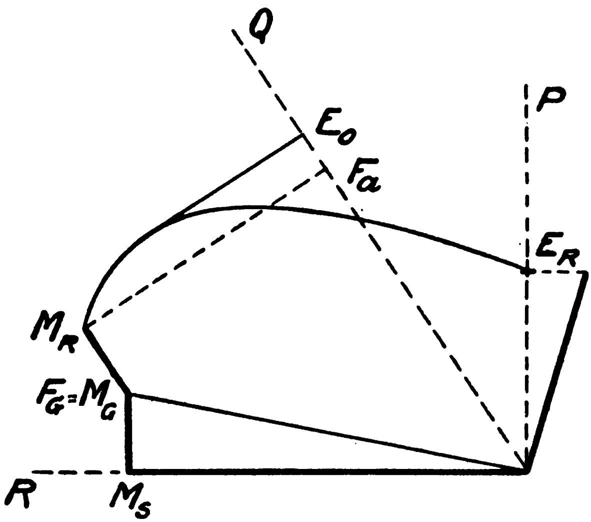


FIG. 68.

OE_r along OP. Since OP is short-circuited, OE_r must be balanced by an air-gap flux producing OE_g and a rotor leakage flux E_gE_r. We can now draw three ellipses (1) the rotor flux ellipse with conjugate diameters OM_r ON_r, where $ON_r = OE_r$; (2) the air-gap flux ellipse with diameters OM_g, ON_g ($ON_g = OE_g$), and (3) the stator flux ellipse with diameters OM_s, $ON_s = ON_g$. Along the axis OR we have the to draw. Projecting it on OQ we get OE_0, the terminal E.M.F., the magnitude of which fixes the scale of our diagram.

E.M.F. $OF_r = OM_r$. Owing to the existence of the flux $ON_r = OE_r$ we have a counter E.M.F. in the rotor equal to $kOE_r = k^2OF_r$. Hence while we have the E.M.F. OF_r along OR in the stator we have only $(1 - k^2)\ OF_r$ in the rotor along the same axis. The importance of this point we shall see later.

The above calculation brings out clearly the analogies and differences of the two types. The stator E.M.F. ellipse in the one is identical with the rotor E.M.F. ellipse in the other, but the existence of such a stator E.M.F. ellipse in the ordinary type involves the existence of a corresponding flux ellipse. In the inverted type, owing to the rotation E.M.F.'s being produced directly in the primary circuit, it involves no such thing, and the flux ellipse remains a straight line at all speeds. Another important difference is the following: Owing to the existence in the short-circuited circuit of an E.M.F. of rotation in phase with the primary current, the secondary current in the ordinary type can get out of phase with the primary and the torque can fall to zero at a certain definite speed. In the inverted motor, on the other hand, this is impossible, and the characteristics of this machine are identical with those of the neutralized series motor, as shown in Chapters I. and II.

The Compensated Repulsion Motor.

This differs from the ordinary repulsion motor only in that the single primary coil is resolved into two, and the coil perpendicular to the short-circuited axis OP is placed on the rotor as shown, by means of an extra pair of brushes.

We saw above that the E.M.F. on the rotor along the axis OR was only $(1 - k^2)$ times that in the stator, and hence by this construction we reduce somewhat the stator E.M.F. required to produce a given current, and since the E.M.F. along OR is in quadrature with the current, we also bring current and E.M.F. more nearly into phase. The effect of this construction on the diagram is shown below.

The flux distribution, etc., is obviously as in the repulsion motor and, in fact, the only variation is in the construction by which we find the stator E.M.F. First, let us re-draw the repulsion motor diagram (Fig. 67) exactly as before, save that the leakage E.M.F. M_gM_s is now parallel to OP like the stator coil (see

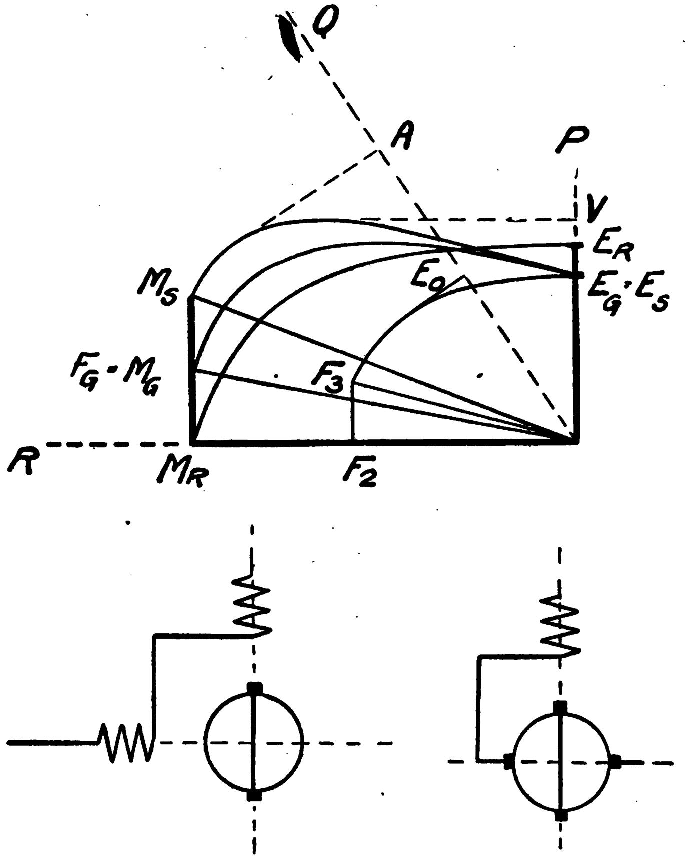


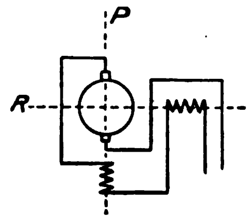
FIG. 69.

Fig. 69). The resultant stator E.M.F. ellipse still exists as before, but the terminal voltage is arrived at in a different manner. The E.M.F. across the stator coil is arrived at as before by projecting this stator ellipse on OP (no longer on OQ, we must note). Call this projection OV.

This E.M.F. has to be added to that on OR in the rotor. This, as we saw above, was $OF_2 = (1 - k^2)\, OM_r$.

In order to compound these we proceed as in Chapter IV. p. 48. Draw a line OQ making an angle $\tan^{-1}\frac{n_1}{n}$ with OP where n is the number of stator turns and n_1 the rotor turns.

Compound OV and OF_2 into an ellipse (note, however, that they are not conjugate diameters, since they are not in time quadrature) and project the said ellipse on OQ.



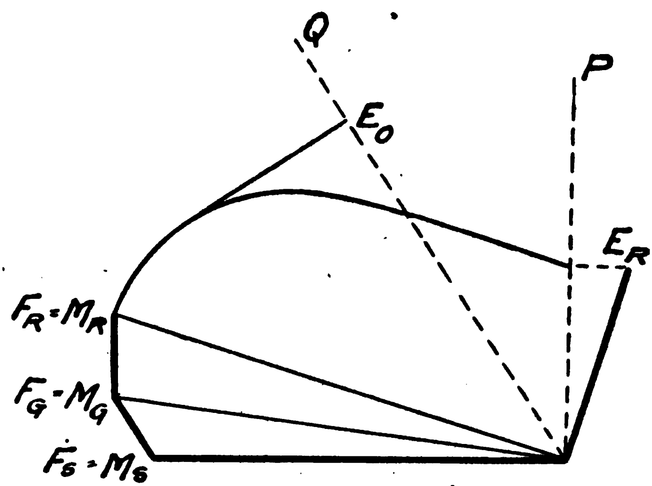


FIG. 70.

Another procedure, shown in the diagram, is to set out F_2 equal to $(1 - k^2)\, F_r$, in accordance with above argument. Add $F_2F_3 = M_gM_s$ the stator leakage E.M.F. to OF_2 and compound with OE_g, which is in quadrature with OF_3, so that OF_3 and OE_g are conjugate diameters, and we can therefore draw the ellipse in a simple manner.

Projecting this ellipse on Q we get OE_0, the terminal E.M.F.

The projection of the stator E.M.F. ellipse on Q is also shown dotted, whence the reduction in the terminal E.M.F. which we obtain by this means is made evident. The improvement in the power factor is shown indirectly also.

We shall now proceed to discuss the neutralized and induced series motors (Figs. 2 and 7) in parallel columns just as we did the ordinary and inverted repulsion motors and shall show that they bear exactly the same mutual relations.

The Induced Series Motor.

Starting. — This machine differs only from the neutralized series motor in that the neutralizing coil is excited from the line. Let OM_r (Fig. 71) be the field flux as usual, and M_rM_g the armature and field leakage flux compounded. We suppose that the armature turns are equal to the primary turns. Set off a line OQ making an angle $\theta = tan^{-1}\frac{n_1}{n}$ with OP, as in the neutralized motor, where n and n_1 have the same meanings. The resultant OM_g of the field and leakage fluxes has to be balanced by that induced from the primary winding. The resultant of these two, in accordance with our rule, will be their projection on OQ, and $OC \times \sqrt{n^2 + n_1^2}$ would also represent the secondary terminal E.M.F. to an appropriate scale if we assume $\sqrt{n^2 + n_1^2}$ to be the resultant secondary turns. This must be balanced by the E.M.F. in the primary

The Neutralised Series Motor.

Starting. — Since in the neutralized motor, armature and neutralizing coil exactly annul one another's M.M.F., there can only be leakage E.M.F.'s along the armature axis OP. Hence, just as in the motors above, we may set off OM_s (Fig. 70) to represent the field flux, M_sM_g to represent the neutralizing coil and field leakage flux compounded, and M_gM_r to represent the armature leakage flux.

coil of n turns. Projecting on OP is equivalent to multiplying by $\cos\theta = \frac{n}{\sqrt{n_1^2 + n^2}}$ and we thus get OD. Hence OD = OC multiplied by the ratio of transformation $\frac{n}{\sqrt{n_1^2 + n^2}}$ between primary and secondary and is therefore the primary induced E.M.F. Adding the leakage flux DE_0 we get the terminal E.M.F. OE_0. The diagram makes it clear that the primary leakage flux might have been added to the secondary leakage flux before projection.

Running at speed k.—We now have an E.M.F. $OE_r = kOF_r$ along the axis OP in addition to those existing before, and the secondary terminal E.M.F. will be the projection on OQ of the ellipse resultant of OF_g and OE_r, which are in quadrature and therefore conjugate diameters. Projecting again on OP we get OD_2, and adding the stator leakage E.M.F. as before we get the terminal E.M.F. fixing the scale of the diagram.

Running at speed k.—We now have an E.M.F. $OE_r = kOF_s$ along the axis OP in addition to those existing before, and the terminal E.M.F. will be the ellipse which is the resultant of OF_r and OE_r projected on a suitable axis. Following the rule of Chapter IV. we set off a line OQ making an angle $tan^{-1}\frac{n_1}{n}$ with OP where n = number of armature turns, and n_1 = number of field turns. We next compound OF_r and OE_r to give the ellipse shown, and project this on OQ to give the terminal E.M.F.

To summarise briefly our discussion of the five series type motors:

Comparing the discussions we have given of each, we see that all the diagrams show four E.M.F.'s, viz.:

(1) The field volts, corresponding to the field flux.

(2) The rotation volts, due to the rotation of the armature in the field flux.

(3) A leakage flux along OP, the armature axis.

(4) A leakage flux along OQ, the resultant axis of the field and neutralizing coils.

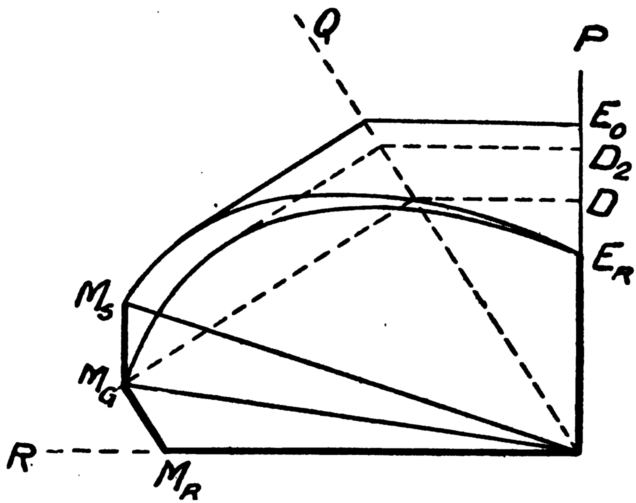


FIG. 71.

These four E.M.F.'s occur in all the motors, but in different motors they appear in different circuits.

In the accompanying table the different circuits in which these four E.M.F.'s occur are given and also the symbols distinguishing them in the diagram. Since there is a flux corresponding to each of these E.M.F.'s except the rotation voltage, and this flux, in accordance with our convention, is represented by an equal vector, we have used the flux and E.M.F. symbols rather indiscriminately. It is, however, easy to replace flux symbols by E.M.F. symbols and *vice versâ* (see p. 66).

Type.	Field Volts.	Rotation Volts.
Repulsion—	*Appear in*	*Appear in*
Ordinary .	Stator or Primary	Transf. to Stator
Symbol .	$M_r = F_r$	E_r
Inverted .	Rotor or Primary	Rotor
Symbol .	$M_s = F_s$	E_r
Compensated	Rotor or Primary	Transf. to Stator
Symbol .	$M_r = F_r$	E_r
Series—		
Neutralized .	Stator or Primary	Rotor
Symbol .	$M_r = F_r$	E_r
Induced .	Stator or Secondary	Transf. to Stator
Symbol .	$M_s = F_s$	E_r

Type.	Leakage along OP.	Leakage along OQ.
Repulsion—	*Appears in*	*Appears in*
Ordinary .	Rotor or Secondary	Stator or Primary
Symbol .	M_rM_g	M_gM_s
Inverted .	Stator or Secondary	Rotor or Primary
Symbol .	M_sM_g	M_gM_r
Compensated	Rotor or Secondary	None. Stator leakage along OP
Symbol .	$M_r = F_g$	M_gM_s
Series—		
Neutralized .	Neutralizing coil	Arm. and field in series
Symbol .	M_gM_r	M_sM_g
Induced .	Stator or Primary	Arm. and field in series
Symbol .	$M_g = F_s$	M_gM_s

From this table one may build up the diagrams very simply.

That these leakage E.M.F.'s must be represented by vectors along the axes of the circuits in which they occur follows from the fundamental principle of the system of space-vectors we employ. Since we have neglected resistance, and since every flux produces an E.M.F. exactly in quadrature with itself, these being the only

E.M.F.'s in the circuit, it is not hard to see that all these fluxes and E.M.F.'s are in phase with one another at starting.

These two points being understood, and the magnitudes of the two leakage fluxes with relation to the field vector (which we assume to have an arbitrary length, say 10 cms.) being supposed known, we have only one more point to decide before we draw our diagram.

In which of the two possible senses is any vector to be drawn?

Perhaps the simplest way to decide this is to remember that by adding leakage E.M.F.'s to an assumed field E.M.F. we can under no circumstances decrease the terminal voltage. The leakage E.M.F.'s should therefore be drawn in the sense which gives the greater terminal volts.

Having decided these points we have simply to compound the three E.M.F. vectors that exist at starting and project them on the axis OQ, drawn in the manner described on p. 48, to get the terminal E.M.F.

If we assume a constant current this diagram remains unaltered during running.

The only difference consists in the addition of a rotational E.M.F. which either occurs directly in the primary circuit or is transferred thereto by transformer action. This E.M.F. must be suitably compounded with the other three to form the new terminal E.M.F.

Owing to the fact that all the fluxes are in phase with one another at starting, the air-gap flux ellipse necessarily reduces to a straight line. The rotation voltage is determined from the following facts:

(1) It is at right angles to the E.M.F. corresponding to the starting air-gap flux in space, and in quadrature with it in time.

(2) It is equal in magnitude to this starting E.M.F. multiplied by $k = \dfrac{\text{speed}}{\text{synchronism}}$.

(3) Its "sense" is such that (in a motor) it increases the terminal E.M.F.

Having drawn this rotation voltage we compound it with the resultant of the three starting voltages to obtain the terminal E.M.F. It is in quadrature with all these E.M.F.'s, and therefore the resultant of the three and the rotation voltage form conjugate

G 2

axes of the ellipse. Having obtained this ellipse we project it on OQ to obtain the terminal E.M.F.

Thus the E.M.F. diagrams of all the different types of motor are identical, and also the starting flux diagrams. But under running conditions the flux diagrams are not identical.

In the neutralized series and the inverted repulsion motors the rotation voltage appears direct in the primary circuit, and no transformer flux is necessary to transfer it thereto.

In the other three motors, however, conditions are different. In the ordinary and compensated repulsion motors the rotation voltage appears in a short-circuited coil. We accordingly find a magnetizing current in existence, and a corresponding flux interlinking this short-circuited circuit, producing therein an E.M.F. which balances the rotation voltage. This E.M.F. is also produced in that portion of the stator circuit which may be assumed to be parallel to OP and consumes a component of the terminal voltage exactly equal to the rotation voltage. It is in reality the "component of primary voltage *consumed* by the field linking OP" which is shown in the diagram rather than the rotation voltage itself. Substantially the same remarks hold for the induced series motor.

In studying the present chapter, the reader may find it useful to study the neutralized series motor first, as the simplest and best known, and obtain a clear understanding of the diagram from it before proceeding to apply the same diagram to the other motors.

CHAPTER IX

EFFECTS OF RESISTANCE, SATURATION, AND THE COMMUTATING COIL

Effect of Resistance.

It is not difficult at the stage we have now reached to take account of the effect of resistance on our diagram. We shall confine the studies of the present chapter chiefly to the repulsion motor, as from what has been said above its application to other types will be fairly evident.

We shall simply re-draw the diagram of Fig. 67 in Fig. 73 and add the effects of resistance to it.

We saw in Fig. 67 that at standstill all the vectors, representing field or leakage E.M.F.'s, were in phase with one another, and reach their maxima at the same time, which we may call time 1. In

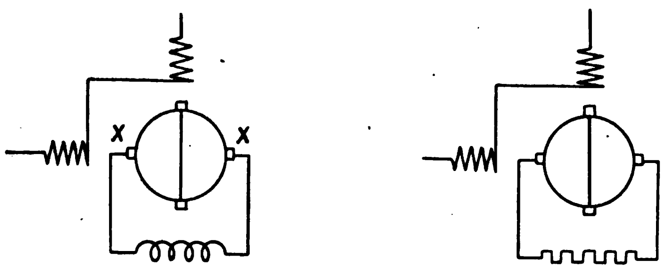


Fig. 72.

accordance with our system of notation explained in Chapter VII., they will therefore be denoted by the symbol F. If now we consider the effects of resistance we shall have a stator resistance E.M.F. parallel to OQ and a rotor resistance E.M.F. parallel to OP; these attain their maxima when T = O, and are therefore in phase with the rotation E.M.F. and denoted by the symbol E. These E.M.F.'s are shown in the diagram as OE_2, the rotor resistance E.M.F. and E_2E_3, the stator resistance E.M.F.

We may now construct the starting flux ellipses for rotor, air-gap and stator. The airgap ellipse is drawn with OE_2 and OF_g

as conjugate diameters and the stator ellipse with OE_s and OF_s as conjugate diameters, the rotor ellipse remaining the same as before. We see that these ellipses are no longer mere straight lines, but are true ellipses, whose area is a measure of the power consumed. The stator ellipse is shown in Fig. 73.

Running Conditions.

During running we have simply to add the rotation voltage OE_r to the voltages at starting and re-draw the ellipses as we did in Chapter VIII. The manner of doing this is fully shown in the diagrams (Fig. 67) so need not be repeated here.

The Effect of the Commutating Coil.

One of the chief disadvantages of many methods of treating the present subject is that they take no account of the effect of the commutating coil, which is, nevertheless, one of the most important secondary effects in most of these machines.

It is clear that a coil commutating under a given brush, as, for instance, the YY brush (Fig. 13), produces a M.M.F. at right angles to that of the circuit through the brush considered. It will therefore have in it the same system of E.M.F.'s as the circuit through another set of brushes perpendicular to the first, such as those shown at XX; this system of E.M.F.'s will be reduced, of course, by the ratio of transformation between the whole armature and the commutating coil; such a commutating coil circuit has a considerable local or leakage inductance and also a considerable resistance, due chiefly to the brush contact. We may, therefore, represent the effects on the field distribution by closing the imaginary XX brushes, by which we replace the commutating coil, through an inductance and resistance in series. We may thus say:—*The effect of the commutating coil on the field distribution of any machine may be represented by replacing the coil by a circuit through a pair of brushes electrically at right angles to that brush under which the coil commutates. This circuit is closed through an appropriate resistance and inductance equal to that of the commutating coil, multiplied by the square of the transformation ratio between the armature and the commutating coil.* If the axis of this equivalent coil is the axis of the field, that is, if there is no field perpendicular to it in

space, and hence no E.M.F.'s of rotation in it, we may place it on the stator instead, when it will appear as a tapping on the field coil, as has been done in Chapter I. (p. 3).

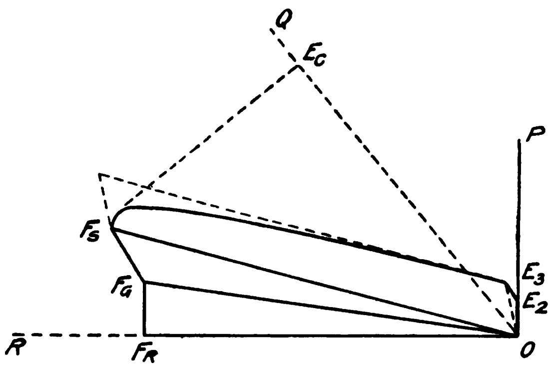


FIG. 73.

This is legitimate for the neutralised series motor, but not apparently for any other type.

Effect of the Commutating Coil in the Repulsion Motor.

We may suppose the circuit through the extra pair of brushes closed through inductance or through resistance and consider these cases separately. The two separate cases are shown in Fig. 72.

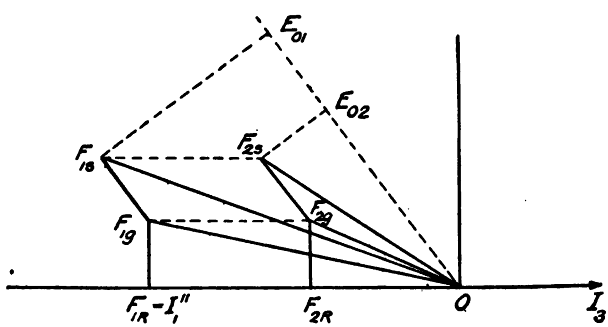


FIG. 74.

Starting Conditions.

(a) *With Inductance only.*

First re-drawing the diagram of Fig. 67, Chapter VIII., let us

consider the effect of the equivalent commutating coil, on the hypothesis of constant current.

If the circuit through the XX brushes is complete, a current I_s (see Fig. 74) will flow in them opposite in phase to the field current I_1''. Its magnitude, of course, depends on that of the inductance in the XX brushes.

The resultant flux OM_{2r} along the XX axis, therefore, and the E.M.F. corresponding OF_{2r} will be that due to the resultant of these two currents, so that $I_1'' - I_s = OF_{2r}$ according to our convention. The two leakage E.M.F.'s are quite unaltered by the presence of the commutating coil, so that we can add these on

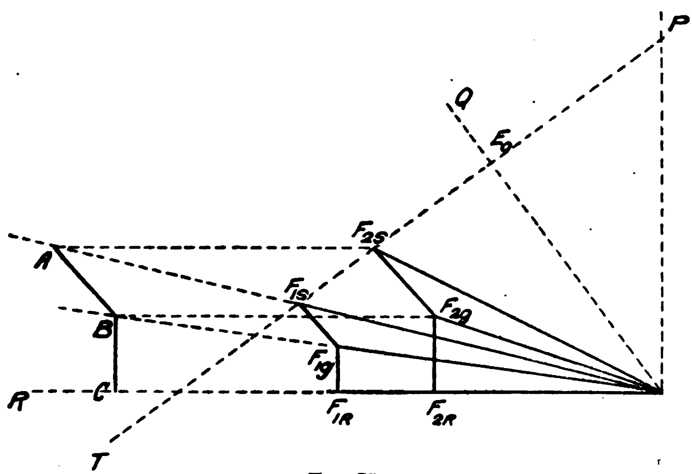


FIG. 75.

as before, obtaining F_{2g} and F_{2s}, the projection of which latter on OQ gives OE_{o2}, the reduced terminal E.M.F. corresponding to the new state of affairs. Accordingly, we see that if the current remains constant the net result of the presence of the commutating coil is to reduce the useful flux along the XX axis, while leaving the leakage fluxes unaltered. Turning now to the more practical case where the terminal E.M.F. remains constant, it is clear that since OE_o, the terminal E.M.F., is fixed, OF_s, the stator E.M.F. ellipse (a straight line), if it varies at all can only move on the line E_oT perpendicular to OQ (Fig. 75).

Suppose F_{1s} is its value when the effect of the commutating coil is zero or the brushes do not bridge any segments, while F_{2s}

is its value when there is a certain commutating coil reaction. We saw above that the effect of this reaction is to exaggerate the leakage E.M.F.'s relative to the field E.M.F. F_{2r}, the leakage E.M.F.'s themselves being independent of the commutating coil reaction on constant current.

Suppose we wish to determine what is the stator current corresponding to the E.M.F. F_{2s}. To do this we proceed as follows:—Produce OF_{1s} OF_{1g} to A and B. Draw lines through F_{2s} F_{2g} parallel to OR to cut the above lines produced at A and B. Then OC will represent the stator current corresponding to OF_{2s} to the same scale that OF_{1r} represents that corresponding to OF_{1s}, while OABC represents the various E.M.F. ellipses which would exist if the machine took the same current OC and had no commutating coil reaction.

It is not hard to see the legitimacy of this construction. For the stator current is proportional to the leakage E.M.F.'s, while in the diagram

$$\frac{OF_{1r}}{OC} = \frac{F_{1r}\,F_{1g}}{BC}.$$

Moreover, if OABC represents the various E.M.F. vectors or ellipses corresponding to the current OC on the hypotheses of no commutating coil reaction, it is clear that in the diagram corresponding to OF_{2s} we have merely reduced the field E.M.F. F_{2r} without altering the leakage E.M.F.'s. This, we saw above, was the effect of the commutating coil on constant current.

(b) *With Resistance only.*

We now come to the case where the commutating coil is closed through resistance only.

In Fig. 76 we repeat Fig. 74, save that the commutating coil current is now in quadrature with OI_1'', and is therefore denoted by OJ_3, and produces an E.M.F. OE_3 in quadrature with itself. To find the E.M.F. ellipses we must compound $OE_3 = OJ_3$ with OF_{1g} as conjugate diameters giving the airgap flux ellipse, and with OF_{1s} giving the stator E.M.F. ellipse, of which the latter is shown in the figure. This investigation, of course, presumes a constant current. To calculate the effects of resistance in the commutating coil directly, on the hypotheses of constant terminal voltage, seems difficult, and we therefore leave it as a subject for future investigation, as also the case in which the commutating coil contains both inductance and resistance.

Saturation.

It is quite easy on the present theory to take saturation fully into account. Its effect is to destroy the proportionality between the current and the flux with its corresponding E.M.F., which we have implicitly assumed by representing both by the same vector. We shall find that its effects on our diagram are much like those of the commutating coil closed through inductance. A similar train of argument leads to a similar result.

Adopting first the hypothesis of constant current, and using the same diagrams as in the case of the commutating coil, where F_{1r}, F_{1g}, F_{1s} represent the rotor, airgap, and stator E.M.F.'s in an *unsaturated* machine in which the field current is I_1''.

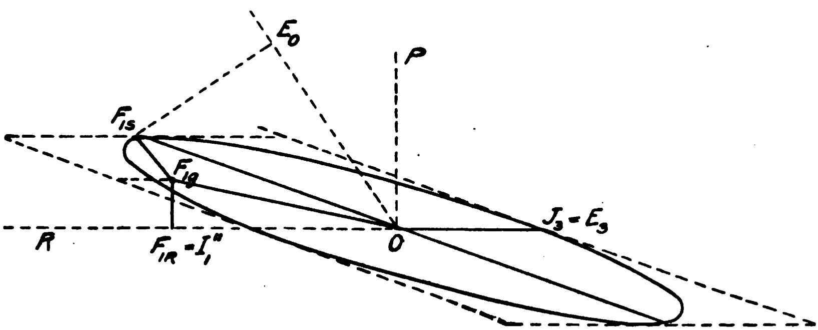


FIG. 76.

Now suppose the main magnetic circuit to be saturated, the leakage magnetic circuits remaining as before.

The main flux and its E.M.F. corresponding is reduced to OF_{2r}, while the leakage E.M.F.'s remain as before. Projecting F_{2s} on OQ, we get OE_{o2}, the terminal E.M.F. corresponding to the new state of affairs.

Thus we see that if the current remains constant, the net result of the presence of saturation is to reduce the useful flux along the axis OR while leaving the leakage fluxes unaltered.

We may now turn to the case of constant E.M.F.

Our argument will be exactly the same as in the case of the inductive commutating coil. Since OE_o, the terminal E.M.F., is fixed, OF_s can only move along E_oT perpendicular to OQ. Suppose F_{1s} is its value when there is no saturation, while F_{2s} is its value when there is a certain saturation.

To determine the stator current corresponding to the E.M.F. OF_{2s} produce OF_{1s} and OF_{1g} to A and B. Draw lines through F_{2s} F_{2g} parallel to OR to cut the above lines produced at A and B. Then OC will represent the stator current corresponding to OF_{2s} to the same scale that OF_{1r} represents that corresponding to OF_{1s}. The legitimacy of this construction may be proved exactly as above.

Running Conditions.

We have now discussed fairly fully the effect of the commutating coil and of saturation during starting, when they are most serious. Both these effects diminish as the speed increases and the field strength and current are reduced. Nevertheless, it is desirable to devote some attention to the running conditions.

Effect of the Commutating Coil in Limiting the Speed of the Repulsion Motor.

In order to make this point clear, it will be advisable to consider the ideal motor first, devoid of magnetic leakage.

This is shown with its diagram in Fig. 77.

We saw above (p. 75) that the rotor E.M.F. along OR was $(1-k^2)$ times the stator E.M.F. along the same axis. This E.M.F., as we saw above, is that existing in the equivalent commutating coil circuit of Fig. 77, which we shall still suppose closed by self-induction. Accordingly, as the motor speeds up from standstill the E.M.F. in the commutating coil goes down very rapidly, firstly because on constant terminal E.M.F. the strength of the field along OR decreases, and secondly because the quantity $(1-k^2)$ rapidly decreases also, becoming zero at synchronism, when $k = 1$. It is for this reason that the effects of the commutating coil at starting are so much more important than at any other speed. At every speed below synchronism, of course, the current in the commutating coil opposes the stator current. Above synchronism, however, the quantity $(1-k^2)$ becomes negative, and the M.M.F. of the commutating coil is reversed and *assists* the stator current. We must now investigate the effects of this.

Let us return to the ideal motor of Fig. 54, supposed now to be fitted with an auxiliary set of brushes equivalent in effect to the commutating coil.

The flux distribution is absolutely determined by the rules

given on p. 51 quite independent of the stator currents; and the flux distribution, of course, determines the magnetizing current distribution. In our previous investigation the magnetizing current along OR was simply the projection of the stator current on OR. Now, however, it is the resultant of this projection and that of the current in the commutating coil.

Since the resultant is invariable, being determined by the invariable flux ellipse, it follows that if the commutating coil current opposes the stator current, the latter must increase to preserve the resultant invariable; this we also saw when investigating commutating coil effects at starting. Also, and this is the point of importance to us at present:—If the commutating coil

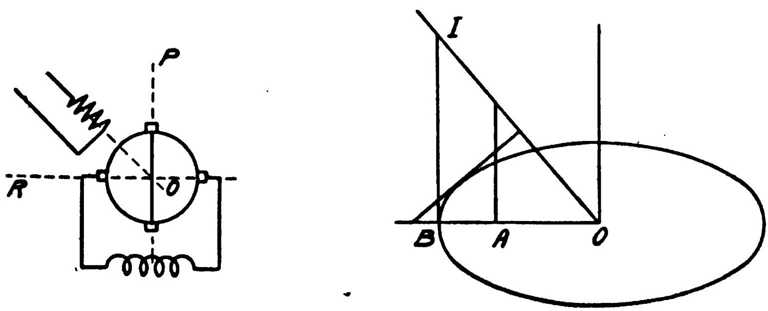


FIG. 77.

M.M.F. assists that of the stator current, the latter will be *decreased*, and if the M.M.F. of the commutating coil is sufficiently great it may even be necessary for the stator current to reverse in order to keep the resultant of the two invariable.

If the stator current reverses, the machine becomes a generator instead of a motor, and hence a machine which is not driven externally will only run up to such a speed that the torque falls to zero just before the generator action commences.

In the diagram if AB represents the commutating coil current (assisting the stator current), then OA will be the component of the stator current along OR. As AB increases OA of course decreases (since OB is invariable), and may finally be reversed.

It is clear that the negative quantity $(1-k^2)$ rapidly increases in value as the speed rises above synchronism when $k = 1$, and in practice it is found that repulsion motors of moderate size, say

from 5 to 10 h.p., seldom rise above 1·5 to 1·75 times synchronism on no load.

The Commutating Pole.

The commutating pole is a device considerably used to improve the commutation of various types of motor during running. It is, however, quite ineffectual during starting, when the effects of the commutating coil are greatest, so that its advantages are strictly limited.

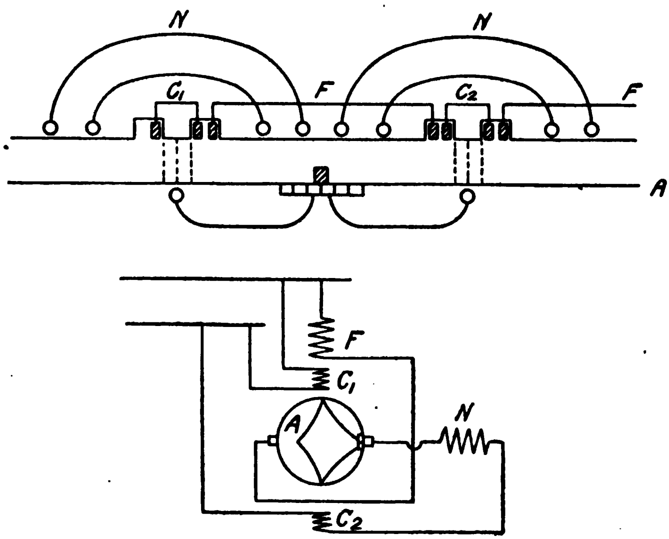


FIG. 78.

The object of the commutating pole is to produce a small local flux which may be cut by the commutating coil alone and not by the rest of the armature. It is accordingly made very narrow circumferentially, spanning one or two slots only. Its action is to induce in the commutating coil an E.M.F. of rotation adapted to annul either the transformer voltage due to the main field in quadrature with the field current, or the reactance voltage.

This rotation voltage, of course, is in phase with the flux to which it is due, and hence if we wish the commutating pole to annul the transformer voltage due to the main field, the commutating pole flux must be in quadrature with the main current. Since

in a well-designed machine the main current will be approximately in phase with the E.M.F., such a commutating pole is often connected in shunt to the line. In this case the flux in it is in quadrature with the line E.M.F., and hence approximately in quadrature with the current, which is what is required.

The commutating pole is also used to cancel the reactance voltage, in which case it is excited by series coils, through which the main current flows in the same way as when used on direct current.

Hence commutating poles are usually excited by both series and shunt coils, the shunt coils being adjustable by hand. In some cases alternate commutating poles are excited with series and with shunt coils. The construction of a machine with commutating poles is shown in Fig. 78, which shows both series and shunt commutating poles. Many different arrangements of commutating pole have been described by various inventors, but it is not the purpose of the present treatise to discuss them.

CHAPTER X

COMPARISON OF THEORY WITH EXPERIMENT—CONCLUSION

WE have hitherto written on the assumption that every quantity has been a pure sine wave, and it is now time to put ourselves on an unimpeachable logical basis in respect of this matter, as it is well known that in existing machines the distribution of the windings is not a sine wave.

Electricians have been long familiar with the "equivalent sine wave" of, say, electric current, which is usually defined as being the sine wave which would give the same ammeter and wattmeter readings as the wave to which it is equivalent. Our theory is a generalization of this. We define an "equivalent" vector ellipse as an ellipse which gives rise to the same ammeter, voltmeter and wattmeter readings on two axes fixed in space as the distribution to which it is equivalent.

In fact, if we take observations of magnitude and phase on two axes only, we get four observations, just enough to give us our ellipse. With observations on two axes only, it is possible to obtain one harmonic in the general Fourier series, representing the space distribution of our quantity, just as with two observations, such as ammeter and wattmeter readings, we have just enough to determine one harmonic in an ordinary alternating quantity and no more. Hence, our representation stands or falls with the "equivalent sine wave" of the ordinary theory. It is the "equivalent sine wave" applied to space distribution. If we reject it, we must also reject the "equivalent sine wave" in time distribution which has been used so long.

It may be remarked that the assumptions which we have used are also characteristic of almost all publications on the subject, although this may not be apparent, through the complications which many of them involve. An easy test may be applied as follows. If the theory in question is entirely free from vectors or from trigonometrical functions of any kind, other than Fourier series, it does not assume a sine wave, otherwise it does. Some

theories, it is true, assume a sine wave in time only and not in space, but even this is not true of the usual treatment.

It may not be out of place to give an illustration of this theory of an "equivalent vector ellipse." Suppose by taking point-to-point observations round the periphery of a machine we determine that the voltage distribution is that of the irregular figure shown in full lines in Fig. 79.

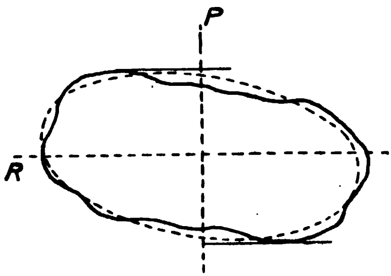


Fig. 79.

Suppose also we take observations of voltage along the two axes OP and OR and of the phase of the two voltages relative to some fixed phase—four observations in all. From these four observations we draw an ellipse by the rules given above. This ellipse will be said to be the ellipse equivalent to the irregular figure. It is obviously the most general figure which we can draw from four observations only.

With these preliminaries we may proceed to the main purpose of the present chapter.

In the preceding chapters we have discussed the theoretical aspect of our vector ellipses fairly fully, so that in the present, and last, chapter we shall endeavour to bring our theory into direct touch with practice, by showing how such vector ellipses may be observed on actual machines and plotted to compare with theory. Let us start as usual with a repulsion motor of the type having two primary coils at right angles. In order to draw a complete series of curves of field distribution, it is only necessary to take the four observations shown in the diagram, viz. the voltages V_1 and V_2, across both stator coils separately V_0, the volts across terminals, and the volts V_3 at standstill across the rotor perpendicular to the short circuited brushes, all these voltages being those corresponding to some particular current. These observations must be taken at standstill and

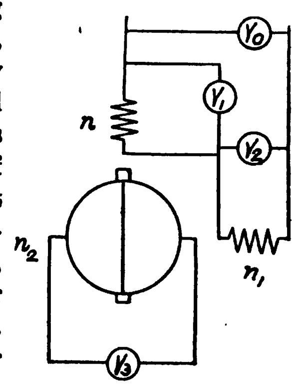


Fig. 80.

also at some speed, k the current being the same in both cases. We are now in a position to draw by direct experiment the diagram we arrived at in Chapter VIII.

(1) Set off V_0, V_1, and V_2 to form an E.M.F. triangle, and resolve V_1 along and perpendicular to V_2, giving V_{1a} and V_{1b}.

(2) Note the turns n and n_1 on the two coils of the repulsion motor and the turns n_2 between brushes on the rotor.

(3) Reduce V_{1a} and V_{1b} to the field circuit by multiplying them by the transformation ratio $\frac{n_1}{n}$.

(4) Set off V_2 and V_{1b}, as reduced, which are in phase with one another, at right angles as shown, giving a triangle OAB.

(5) Reduce V_3 to the stator circuit by multiplying by the transformation ratio, giving

$$V_{3a} = V_3 \frac{\text{Stator condrs. } (n_1 \times 2) \times \text{Rotor circuits}}{\text{Rotor condrs.} \times \text{Stator circuits}}.$$

At starting V_{3a} will only be slightly less than V_2.

From the point B set off a line making an angle $\tan^{-1} \frac{n_1}{n}$ with AB.

From the point V_{3a} draw a line parallel to AB to meet the line BC in C. Draw the line OC.

(6) Now set off from O perpendicular to OA the voltage V_{1a}, as reduced, which is in quadrature with V_2, giving OD.

Then OB and OD are conjugate diameters of the stator E.M.F. ellipse, which we can immediately proceed to draw, and OC and OD are conjugate diameters of the airgap flux ellipse.

This construction is applicable at all speeds or at standstill. In the latter case OD represents resistance effects, while, when the machine is running, it represents the counter E.M.F. as well.

We may set off V_0 along OQ and project the stator ellipse on OQ as a check. This projection should, of course, equal V_0.

If we keep the current constant at all speeds by varying the terminal E.M.F. it will not be necessary to observe V_3 for more than one case (standstill).

By means of such observations as these the field distribution of any type of motor may be experimentally determined. The

reader who is interested in some particular type of motor will not have much difficulty in devising a suitable set of observations for plotting the field distribution.

By their means also a number of the secondary effects in the motor, notably the effects of saturation and the commutating coil, may be separated. The effect of the commutating coil at starting may be separated as follows. First take observations as above on the motor fitted with brushes in the usual way. Secondly, repeat the observations with the secondary circuit closed through contacts arranged on the commutator so as not to bridge the segments. Comparing these two sets of observations, we can separate the effect of the commutating coil.

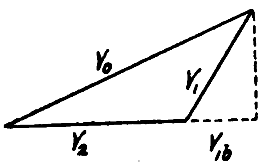


FIG. 81.

Saturation.—If it were not for saturation it is clear that the effect of increasing the terminal voltage would merely be to

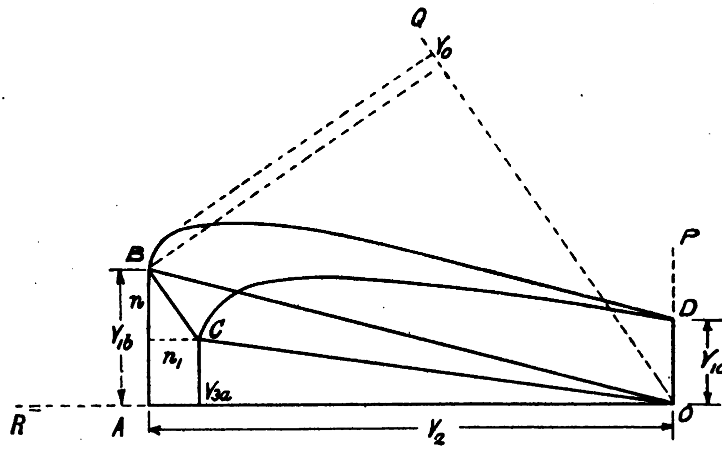


FIG. 82.

increase all the vectors of the diagram (Fig. 82) in the same proportion. Apart from scale, therefore, the diagram would be the same for all voltages.

If, therefore, we draw the diagram for several different voltages, any distortion which may occur on the higher voltages can only be due to, and is a measure of, the saturation. The effect of

saturation may therefore be separated out by drawing diagrams based on observations taken at several different terminal voltages, and plotting them to such different scales that the varying terminal voltages are always represented by the same length.

Before concluding we must introduce a caution with regard to the plotting of ellipses of E.M.F., etc., from point-to-point observations, which is absolutely essential if we are not to fall into hopeless confusion.

The natural way to plot these observations would be the following :—

Suppose we have a 2-pole drum-wound machine having, say, only 16 coils on it, as shown in Fig. 83. Take observations of the voltage (R.M.S.) across each coil, and set it out along an axis OR perpendicular to the plane of the coil.

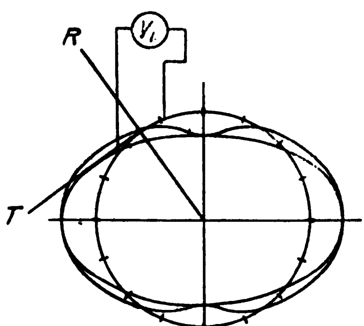


FIG. 83.

What result will such a procedure give us? Suppose the flux threading the coils and its accompanying E.M.F. to be elliptical in form. We saw in Chapter III. that the maximum (and therefore R.M.S.) E.M.F. or flux along an axis is given by the projection of the flux ellipse on that axis.

Hence in Fig. 83 the voltage V_1 across any coil is found by projecting the ellipse on a line OR perpendicular to its plane.

Projecting the ellipse on a number of equidistant radii in the manner shown, and drawing the curve of intersections of these radii with tangents perpendicular to them, we get, not an ellipse, but the pedal curve of an ellipse with respect to the point O, a curve with two lobes touching the ellipse at the extremities of its axes. When we have observed this pedal curve, we can, of course, easily determine the axes of our ellipse, which are the axes of symmetry of our curve. When we know the axes we can draw the ellipse by the trammel method or otherwise. If, however, we do not wish to assume that our flux distribution, etc., is an ellipse, we can find out its actual nature by reversing the con-

struction by which we drew the pedal curve. The curve enveloped by the tangents is then the curve of flux distribution.

Such ellipses of E.M.F. could only be observed directly by connecting an oscillograph to a synchronously revolving contact maker, which would exactly follow the radius vector of the ellipse as it goes round the curve. The intercept of the ellipse on any axis does not, of course, represent the maximum E.M.F. on that axis, but merely its instantaneous value at the moment the radius vector passes the axis.

The object of the present treatise may now be said to be accomplished. We have developed from first principles a direct method of investigating the distribution of flux, current, and E.M.F. in single-phase motors which never loses sight of physical principles. We have discussed fairly fully all those secondary effects that make all the difference between a good machine and a bad one, and have brought our theories into direct touch with practice, by showing how the diagrams to which they lead may be plotted from experimental observations. Much, of course, remains to be done in amplifying our investigations, applying them to new types of machine, etc.; but enough, it is believed, has been accomplished to show that the method of the present treatise is a useful weapon of research, leading directly and without circumlocution to just those results which the designer requires. Accordingly, at this point we leave the matter, in the hope that other writers may take it up, and fill any gaps which may be found in the present treatise.

APPENDIX

Reference Propositions in the Geometry of the Ellipse

Prop. 1.—Five conditions are requisite to determine an ellipse. These may take an unlimited number of forms, such as

(a) Five points.
(b) Five tangents.
(c) Two pairs of conjugate diameters* and one point or one tangent.
(d) One pair of conjugate diameters and three points or three tangents.
(e) The foci and one point or one tangent (the foci each count as two conditions).
(f) The centre and three points or three tangents (the centre, also, counts as two conditions).

By means of the propositions which follow, a set of conditions of one kind may in most cases easily be converted into a set of another kind.

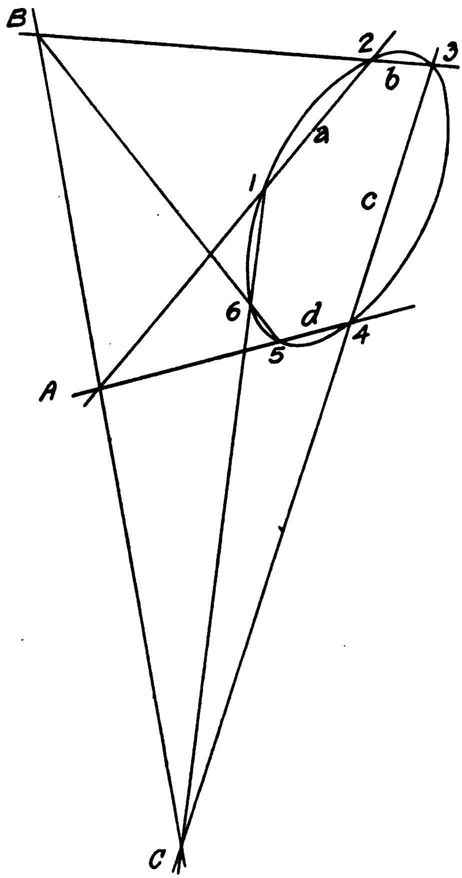


Fig. 84.

* A diameter here means merely a line of indefinite length passing through the centre. Proofs of these propositions will be found in Reye's "Projective Geometry" (Macmillan), to which the writer is indebted for most of his information on the geometry of the ellipse.

All the ellipses with which we shall deal will be concentric, their centre being that of the machine under study; so that, actually, only three independent conditions will be necessary to determine them. A fourth is required, of course, to determine the position of the phase-arrow, making in all the four which we saw above were necessary to determine a vector ellipse completely.

Prop. 2.—Pascal's theorem. The three pairs of opposite sides of any hexagon inscribed in a conic intersect in three points which lie on one straight line (Fig. 84).

By the "opposite" sides of an irregular hexagon are meant the first and fourth, second and fifth, and third and sixth.

Prop. 3.—Brianchon's theorem. The three straight lines joining opposite corners of a hexagon circumscribed about a conic meet in a point. As before, by "opposite" corners are meant the first and fourth, second and fifth, and third and sixth. The "corners," or vertices, of course, are the intersections of consecutive sides (Fig. 85).

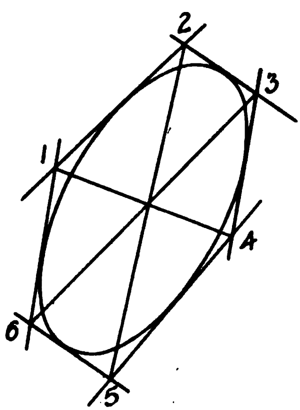


FIG. 85.

We have just seen that five points or five tangents are sufficient to determine an ellipse. The principal use of Pascal's and Brianchon's theorems to us will be to enable us, when we have five points, to draw the tangents at these points, or when we have five tangents, to determine their points of contact with the curve. They will also enable us, when we have five points or tangents, to draw an unlimited number of further points or tangents and so outline the curve.

If in a hexagon inscribed in a conic two adjacent corners or vertices approach indefinitely near to one another, the side joining them assumes the position of a tangent to the curve and our hexagon reduces to an inscribed pentagon and a tangent at one of its vertices (see Fig. 86).

Similarly, if in a hexagon circumscribed to a conic two adjacent sides approach indefinitely near to each other, in place of the vertex in which they intersect we have a point of contact and hence our circumscribed hexagon becomes a circumscribed pentagon with one point of contact with the curve (see Fig. 87).

When the sixth side of the hexagon vanishes, we can no longer speak of "opposite" sides of the pentagon which remains. Instead of "opposite" we must read "non-adjacent," as a slight examination of Figs. 86 and 87 will show.

With the aid of the above remarks we may immediately deduce from Pascal's theorem the following construction:

Prop. 4.—Given five points on a conic, 1, 2, 3, 4, 5, to draw the tangents at these points.

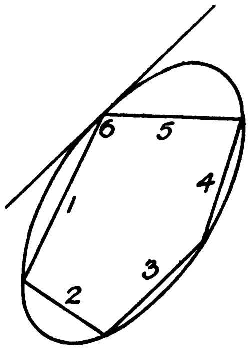


FIG. 86.

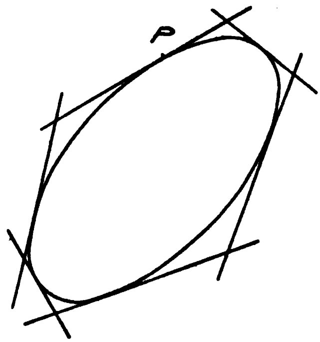


FIG. 87.

Find the point of intersection of the straight line joining 1, 2 (Fig. 88) with that joining 3, 4. Call it A. Find the point of intersection of the

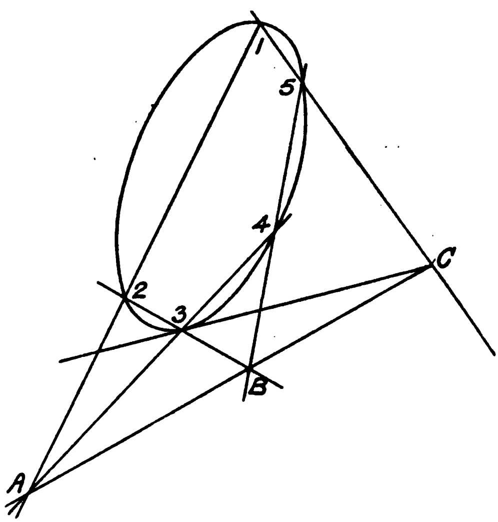


FIG. 88.

straight line joining 2, 3 with that joining 4, 5. Call it B. Draw a line through AB and find the point of intersection of the remaining side 5, 1 with AB. Call it C. Draw the line C 3. This will be the

tangent at the point 3. The point 3 may, clearly, be any of the five points, so we may draw the tangents at any of them by this method.

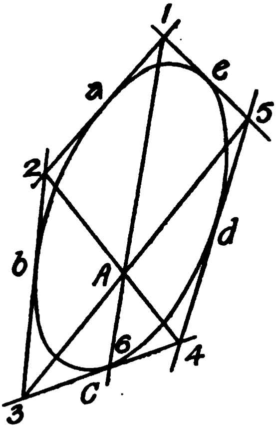


FIG. 89.

Prop. 5.—Given any five tangents to a conic to find their points of contact with the curve.

To find the point of contact of the tangent *c*, for instance (Fig. 89), we join the point of intersection 2 of the tangents *ab* to the point of intersection 4 of *cd* and the point of intersection 5 of *de* to the point of intersection 3 of *bc*. Let A be the point of intersection of the lines so drawn. Now join the remaining point of intersection 1 of *ae* to A and the intersection of this last line with the tangent *c* at 6 will be the point of contact of *c* with the curve. Similarly, we may find the points of contact of the other tangents.

Other important propositions are the following, whereby, having five points or five tangents, we may draw an unlimited number of further points or tangents.

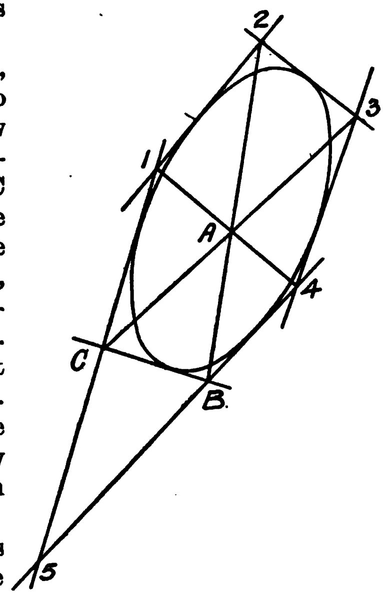


FIG. 90.

Prop. 6.—Given five points, 1, 2, 3, 4, 5, on a conic (Fig. 84) to determine another point 6. Draw the lines joining 1, 2 and 4, 5 to intersect in A. Draw *any* line ABC passing through A and find the points of intersection of the line joining 2, 3 and of the line joining 3, 4 with it. Call these B and C respectively. Join B to 5 and C to 1. Then the intersection 6 of these last two lines will be a point of the curve.

By repeating this construction, one may clearly find any number of new points, simply varying the direction of the line ABC through A.

A similar proposition enables us to draw a sixth tangent, when five are given.

Prop. 7.—These five tangents will intersect in the five vertices shown in Fig. 90 as 1, 2, 3, 4, 5. Join the vertices 1 and 4. Choose any

point A upon the line as drawn. Join 2 and A and 3 and A and find the intersections B and C of these lines with the lines 4, 5 and 5, 1 respectively. The line BC will be a tangent to the curve. By varying the point A any number of tangents may be drawn.

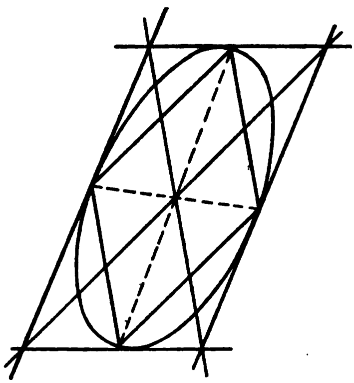


FIG. 91.

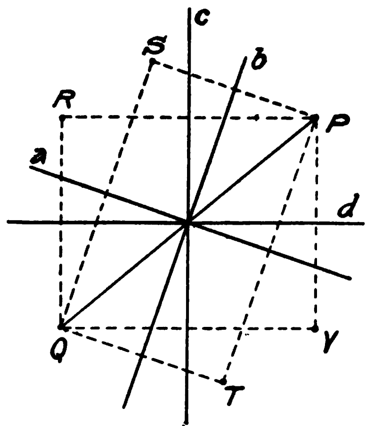


FIG. 92.

Prop. 8.—The diagonals of any parallelogram circumscribed to a conic are conjugate diameters of the curve.

Prop. 9.—The sides of any parallelogram inscribed in a conic are parallel to a pair of conjugate diameters.

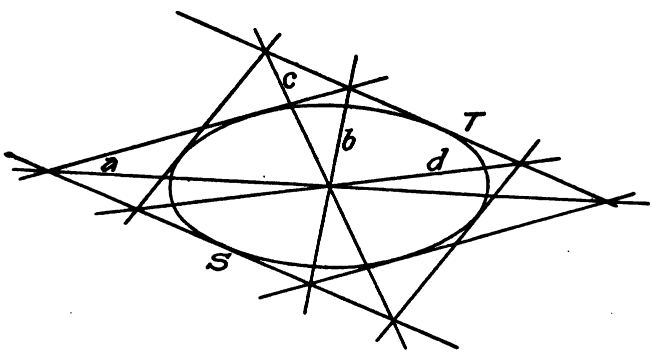


FIG. 93.

If we have a parallelogram circumscribed to an ellipse (Fig. 91) and an inscribed parallelogram whose sides are parallel to the diagonals of the circumscribed parallelogram, then the vertices of the inscribed will be the points of contact of the circumscribed parallelogram.

This is a very simple construction for finding the points of contact of a conic with the circumscribed parallelogram, if we know one of them.

Prop. 10.—Given two pairs of conjugate diameters, *ab* and *cd* and one point P to determine any number of further points (Fig. 92).

Draw the diameter passing through P and find its second intersection Q with the curve from the fact that PQ is bisected at the centre. With PQ as diagonal, construct two parallelograms, the sides of each parallel to one of the given pairs of conjugate diameters.

The two new pairs of vertices, RT and SV, of the parallelograms so determined lie upon the curve. This process may then be repeated.

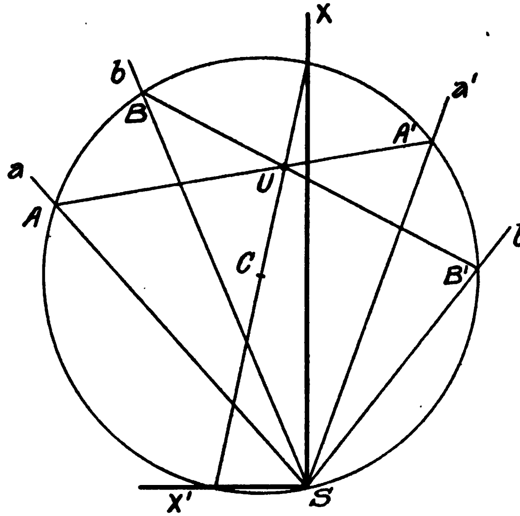


FIG. 94.

Prop. 11.—Given two pairs of conjugate diameters, *ab* and *cd* and one tangent T, to find any number of further tangents (Fig. 93).

Draw another tangent S at the opposite end of a diameter from T and complete the two parallelograms whose diagonals are *ab* and *cd* respectively, and two of whose sides are S and T. This gives four new tangents, with the aid of which we may repeat the above process.

It should be noted that these propositions relating to parallelograms are peculiarly important to us, as, since the centre of our ellipse is always known, two points or two tangents only are sufficient to determine a parallelogram. They are both simpler and give more rapid results than the more general propositions deduced direct from Pascal's and Brianchon's theorems.

Prop. 12.—To determine the major and minor axes from two pairs of conjugate diameters, aa' and bb'.

Through the centre S of the curve, in which the conjugate diameters intersect, pass an arbitrary circle cutting one pair of conjugate diameters in AA′ and the other in BB′ (Fig. 94). Join AA′ and BB′, and through U, their point of intersection, draw a line passing through the centre of the circle. This will cut the circle in XX′. The lines SX and SX′ are now the axes which we seek. Note that since XX′ is a diameter of the circle, the axes are at right angles.

Prop. 13.—To find the foci when the axes, a tangent, and its point of contact with the curve, are given.

Let A be the tangent (Fig. 95), p the point of contact, and C the minor axis. Draw a line perpendicular to A through p cutting C in a. Let A cut C in b. Draw a circle through abp (a semicircle on ab) cutting the major axis in ef. Then ef will be the foci.

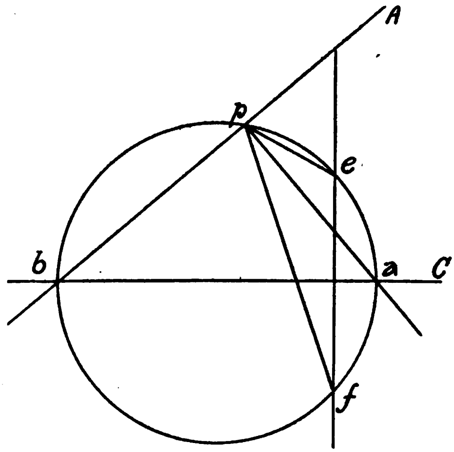


FIG. 95.

To find the length of the major axis.

Note that $pf + ep$ is equal to the length of the major axis.

By striking an arc from one of the foci of radius equal to half the major axis and cutting the minor axis, we get g, h, the extremities of the minor axis.

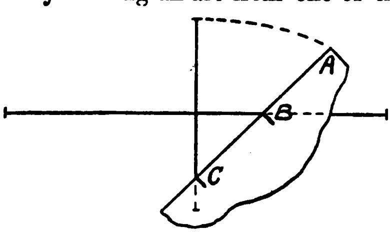


FIG. 96.

Prop. 14.—One of the most convenient practical methods of plotting an ellipse point by point is the well-known trammel method, which presupposes a knowledge of the axes and their lengths.

First draw the axes (Fig. 96), then set off on a separate piece of paper, the major axis AC and the minor axis AB. If we move the separate piece of paper or trammel so that B moves along the major and C along the minor axis, then A will describe the ellipse.

Most of the above purely geometrical propositions relate to the

ordinary ellipse where no considerations of phase arise. The vector ellipse requiring a phase-arrow to complete its specification will be discussed below.

Addition of Vector Ellipses.—If we have two vector ellipses which we require to add, we may either add corresponding radii vectorially at every point and so plot out the resultant ellipse, or we may draw a pair of corresponding conjugate diameters in each ellipse and add these vectorially. This will give a pair of conjugate diameters of the resultant ellipse and their lengths, *i.e.*, two points on each whence we can draw the ellipse by the constructions given above.

For instance, let ab and $a'b'$ (Fig. 97) be a pair of conjugate diameters of each ellipse and let them correspond to the same instants, *i.e.*, a to a' and b to b'. Let $a + a' = a''$ and $b + b' = b''$ when added vectorially. Then a'' and b'' will be conjugate diameters of the resultant ellipse.

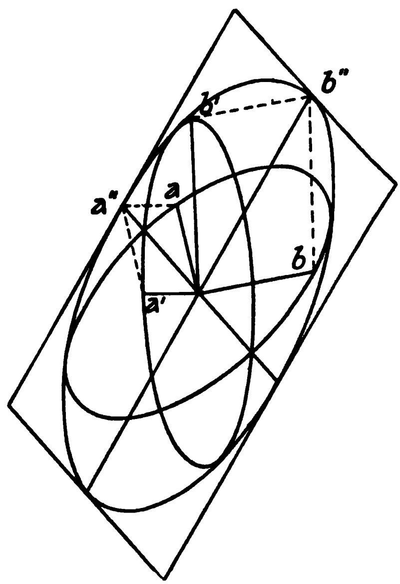


FIG. 97.

Having a'' and b'', we can clearly, from the definition of conjugate diameters, draw at once a parallelogram circumscribing the ellipse, the four extremities of the two conjugate diameters being its points of contact. This will be sufficient to enable us to sketch in the resultant ellipse with very fair accuracy. If further accuracy is desired, we can draw more tangents by Prop. 11, or find the major and minor axes by Prop. 12, since we now have four conjugate diameters, the original pair and the diagonals of the parallelogram. We could then find the foci by Prop. 13, and then apply the trammel method of plotting the ellipse.

It will be found, however, that once in possession of five or six tangents, particularly when their points of contact are known, an ellipse can be sketched in with an accuracy hard to beat by a regular construction. It is very seldom, in fact, that these complete constructions are required.

The Multiplication of Vector Ellipses.—In order to calculate several quantities which will be of importance to us, such as the torque due to

an ellipse of flux and an ellipse of current, or the power corresponding to an ellipse of E.M.F. and an ellipse of current, it will be necessary to investigate the multiplication of vector ellipses.

For purposes of multiplication, we may resolve each ellipse into two oscillating vectors along conjugate diameters and in quadrature with one another. Let the two conjugate diameters of one ellipse be ab and those of the other $a'b'$ (Fig. 98). Now a' is in quadrature with b and b' with a, and hence the mean product of $a'b$ and ab' is zero over a complete period. We have, therefore, only to deal with aa' and bb'.

Now, considering each of the conjugate diameters as an independent oscillating vector representing, say, a flux distribution for a and b, and a current distribution for a' and b', then the torque due to a and a' will be proportional to the projection of a' perpendicular to a. For a flux and a current having parallel axes in space will produce no torque, but merely an attraction or repulsion. Hence the torque due to a and a' will be proportional to $aa' \sin \theta$, θ being the angle between them. It is, in fact, proportional to the area of the parallelogram constructed on aa' as sides which is, of course, $aa' \sin \alpha$.

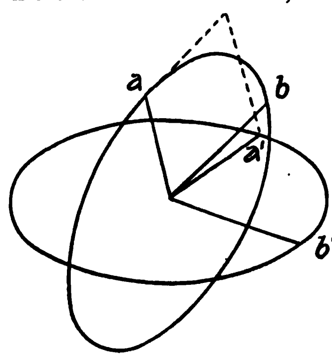


FIG. 98

Similarly, the torque to bb' is $bb' \sin \beta$, being the angle between b and b'.

The product now under discussion will be called, for the present, the sine-product.

Hence we have:

The sine-product of any two ellipses whose corresponding conjugate diameters are ab, $a'b'$, respectively, the angle between aa' being α and that between bb' being β is $aa' \sin \alpha + bb' \sin \beta$.

Similarly, we may define a cosine product which we shall require in discussing the power due to an elliptical polyphase distribution of E.M.F. impressed on a circuit in which flows another elliptical distribution of current. In such a case, if ab are the conjugate diameters of the E.M.F. ellipse, and $a'b'$ of the current ellipse, then $aa' \cos \alpha + bb' \cos \beta$ will be the power.

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